BME 50500: Image and Signal Processing in Biomedicine

Lecture 6: Introduction to Ultrasound

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Content (Lecture Schedule)

Linear systems in discrete time/space
Impulse response, shift invariance (4)
Convolution (4)
Discrete Fourier Transform (3)
Power spectrum (7)

Medial imaging modalities
MRI (2)
Tomography, CT, PET (5)
Ultrasound (8)

Engineering tradeoffs
Sampling, aliasing (1)
Time and frequency resolution (3)
Wavelength and spatial resolution (9)
Aperture and resolution (9)

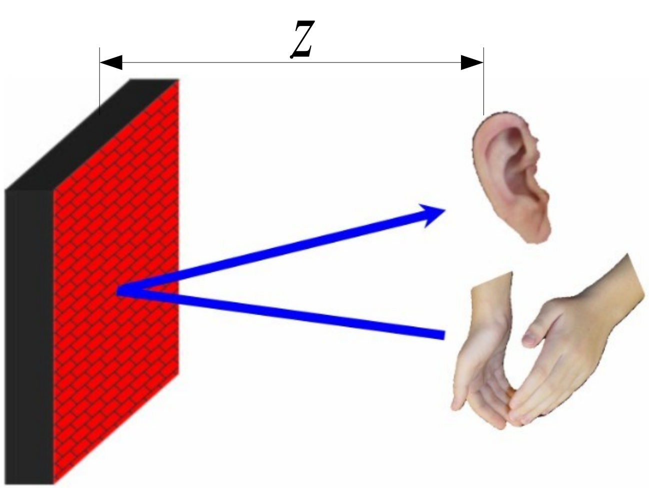
Filtering
Magnitude and phase response (6)
Filtering (6)
Correlation (7)
Template Matching (10)

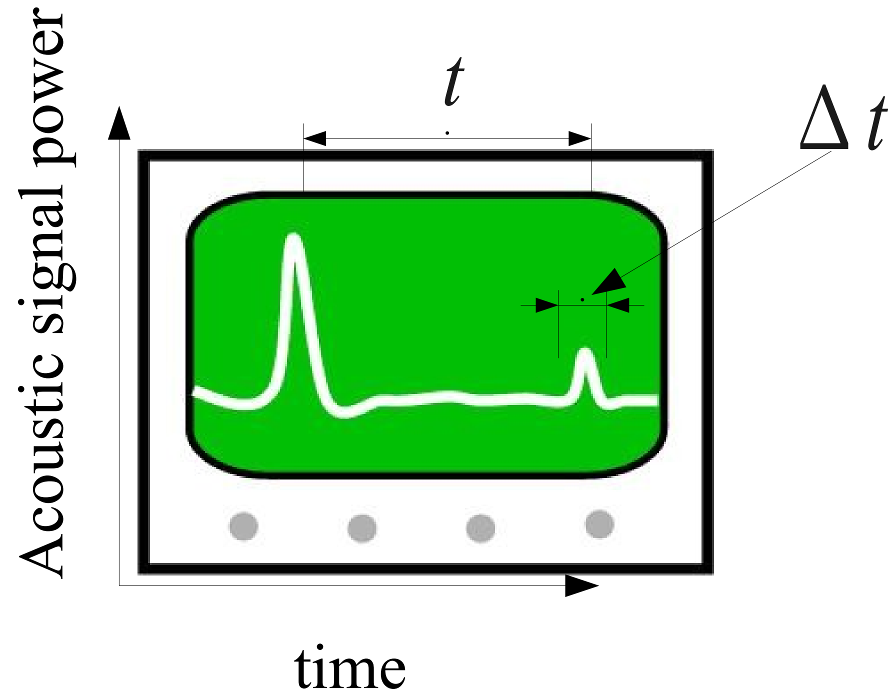
Intensity manipulations
A/D conversion, linearity (1)
Thresholding (10)
Gamma correction (11)
Histogram equalization (11)

Matlab

Basic principle

Ultrasound generates a sound pulse and measure the delay of the echo to determine the distance of a reflector.



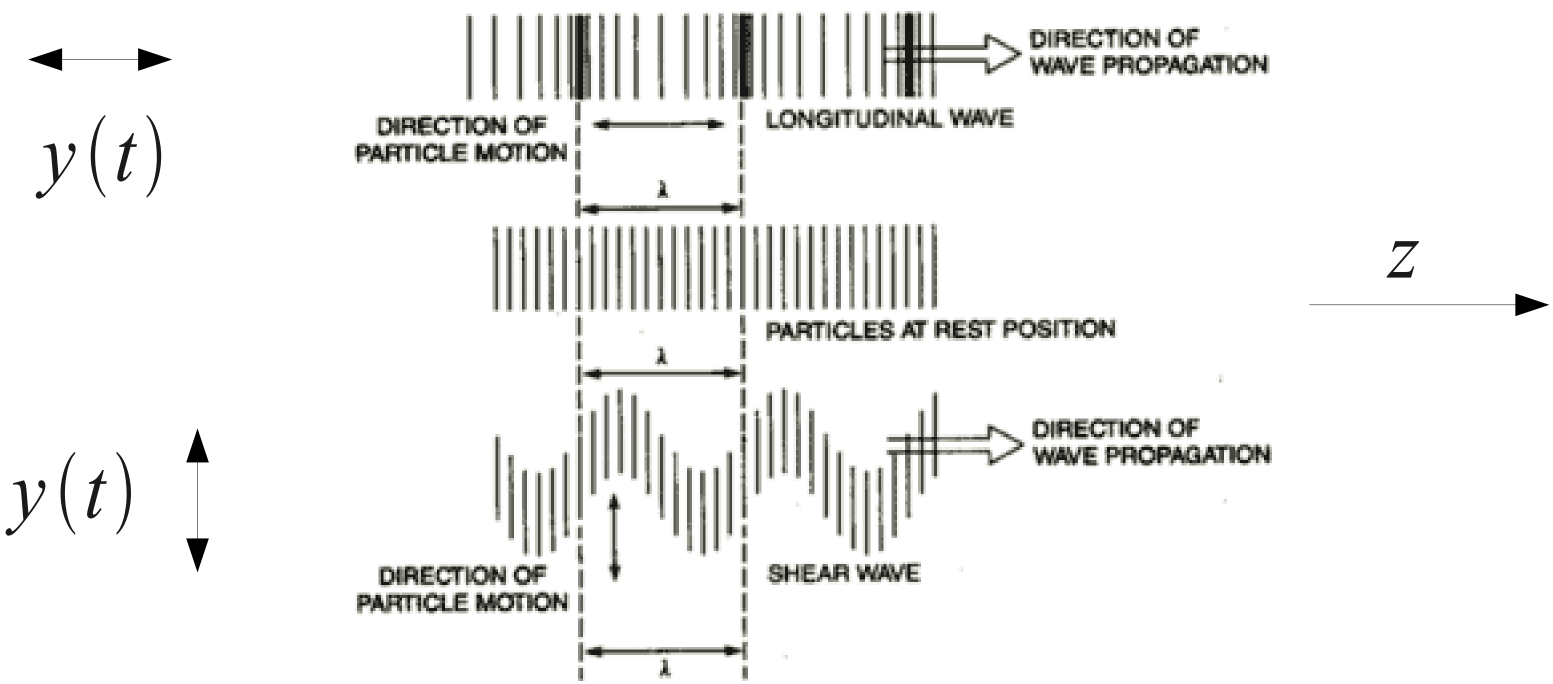


This simplest for of ultrasound is called A-mode giving only depth information:

$$z = v\frac{t}{2}$$

Sound propagation

Sound propagates primarily as longitudinal waves or sheer waves



Displacement $y(t, z) = A \sin(\omega t - kz)$

$$k = 2\pi/\lambda$$

$$\omega = 2\pi f$$

Sound propagation

Velocity = frequency * wavelength

$$v = f\lambda$$

The velocity of sound propagation depends on material properties

- density ρ and
- elastic constant C
  (Young's modulus, and Poisson's ratio)

$$v = \sqrt{\frac{C}{\rho}}$$

Wave travels smoothly in an uniform medium (constant velocity).

Non-uniform medium (varying velocity) leads to

- Reflection
- Scatter

Depth resolution

The resolution of depth will depend on the wavelength:

$$\Delta z = \frac{\Delta t}{2} v \propto \lambda$$

Resolvable distance is proportional to the wavelength. This is true for almost all imaging methods either acoustic, optic of any other electromagnetic radiation!

$$\lambda = v / f$$

Shorter wavelength and higher frequencies give higher resolution, hence **ultra**sound.

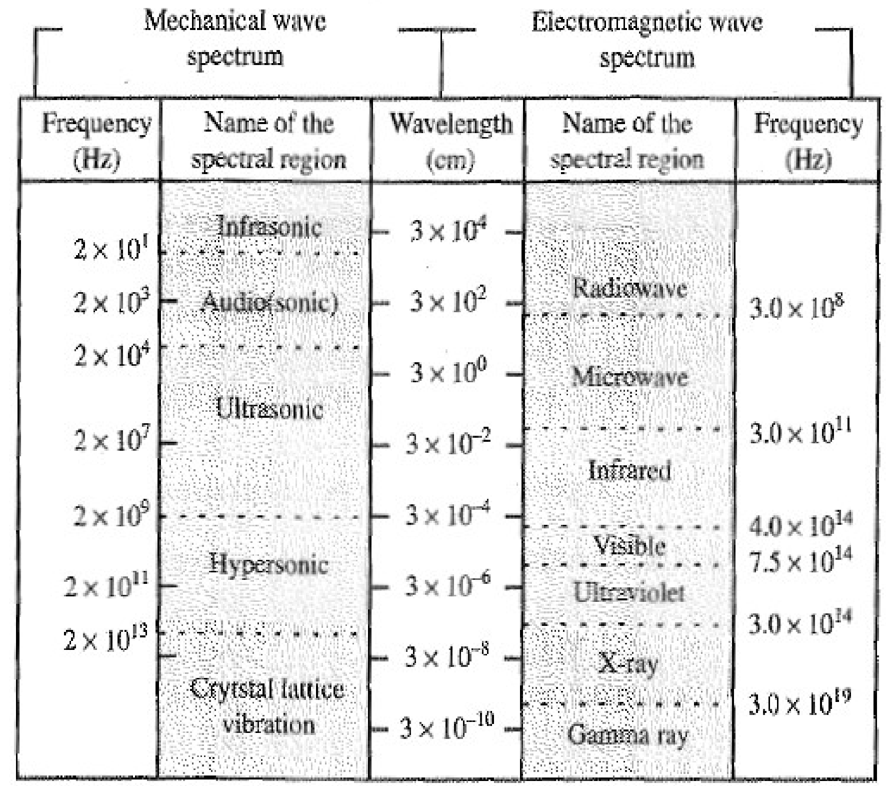


Fig. 2.2 Mechanical and electromagnetic wave spectrum

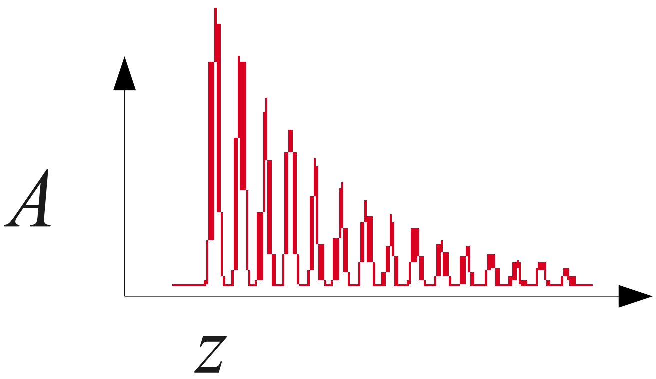
Raj, Rajendran, Palanichamy, “Science and Technology of Ultrasound”, 2004

Attenuation

As the sound travels through the material however it is attenuated by absorption and scatter. This can be approximated by exponential decay of the amplitude of the signal:

$$A(z) = A_0 e^{-z/z_e}$$

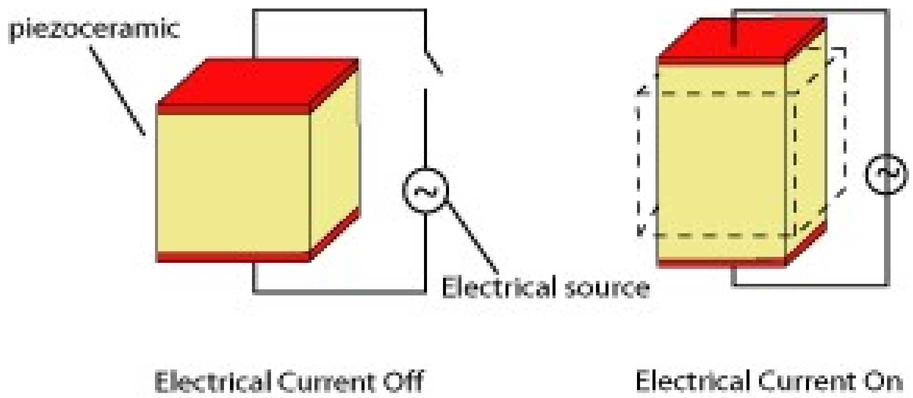
$$y(t) = A \sin(\omega t - k z)$$

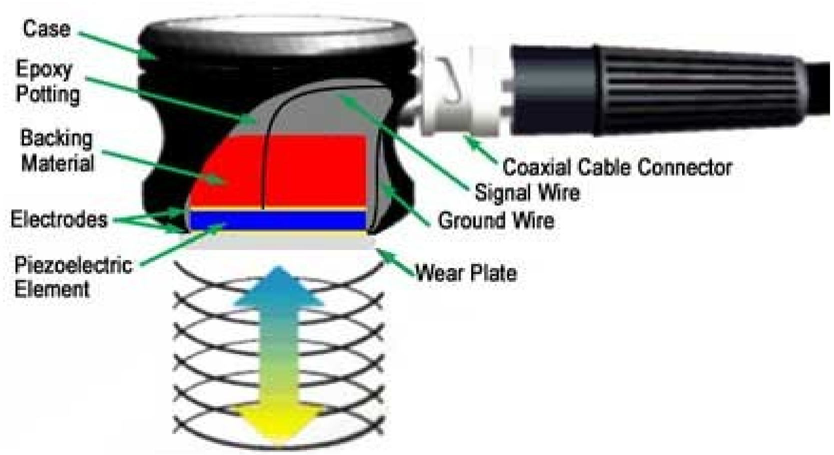


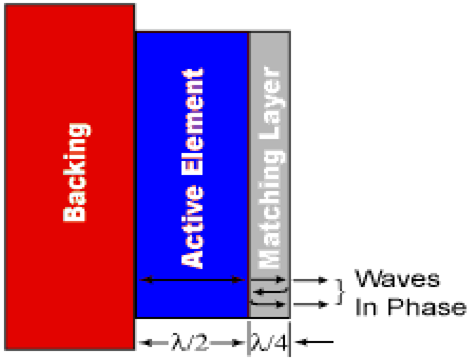
When the strength of the reflected signal is displayed it needs to be corrected for this exponential decay.

Transducer

Sound is generated and measured with Piezoelectric elements.

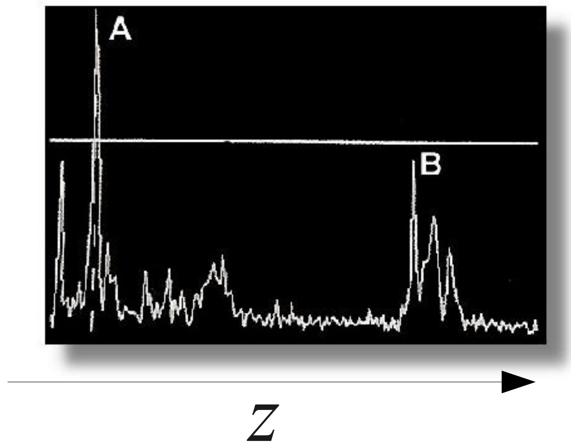


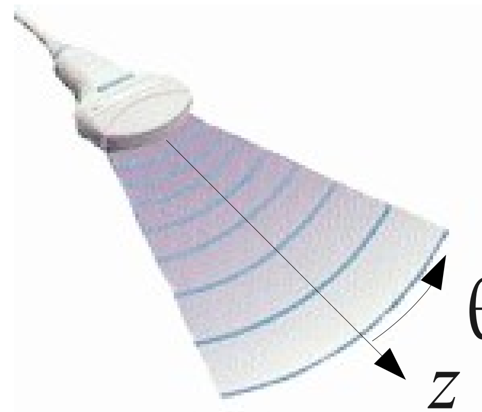




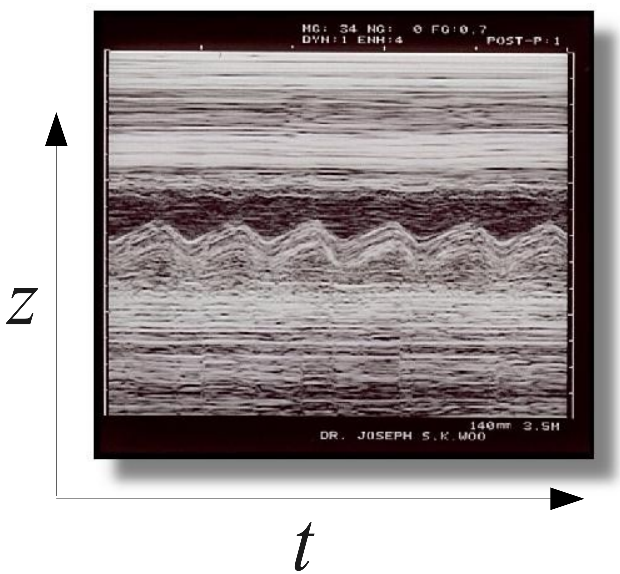
Ultrasound modes

A-mode

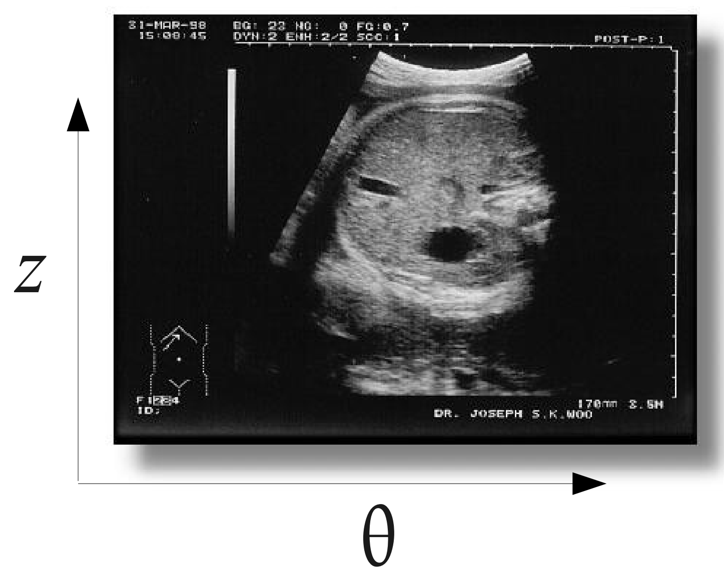




M-mode (A-mode repeated in time)

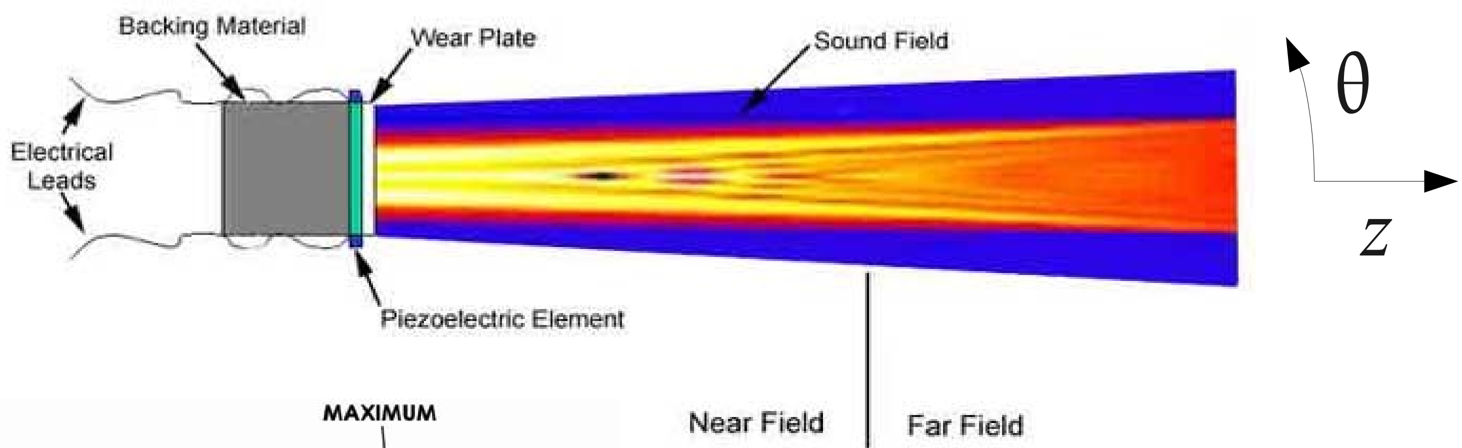


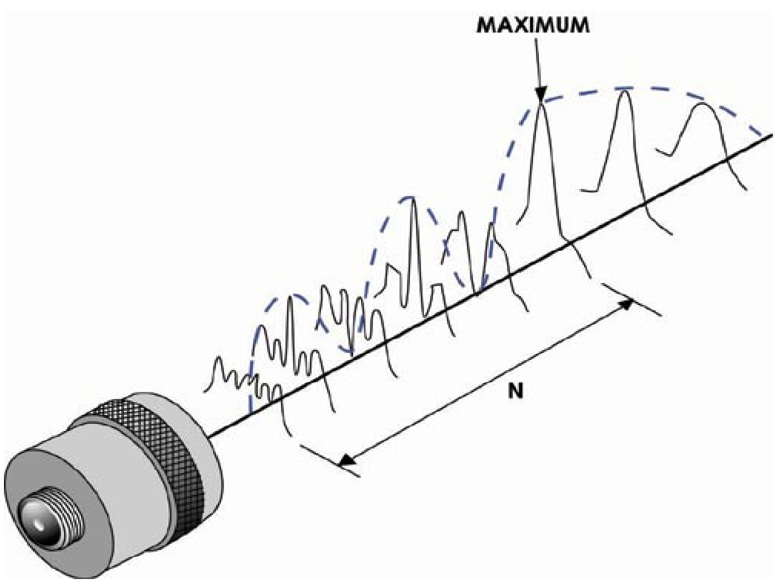
B-mode (A-mode repeated for different 'beam' directions)



Beam for transverse resolution

B-mode requires a narrow beam which gives resolution in the transverse direction θ.

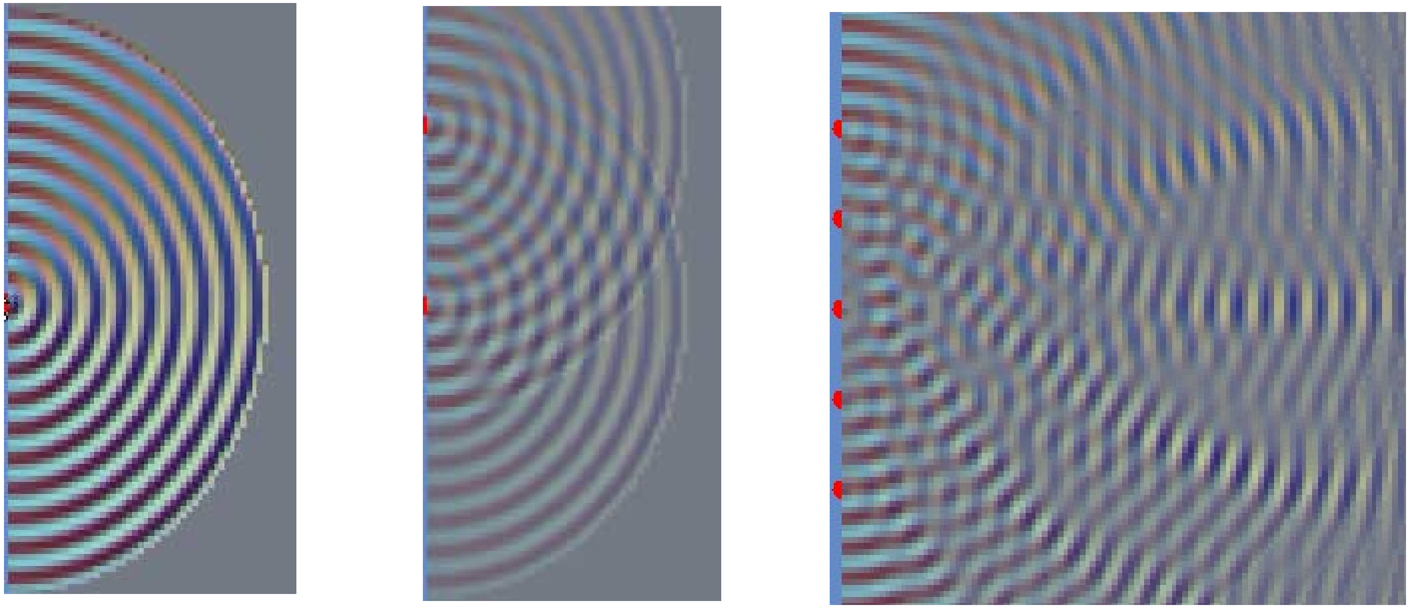




Many of these figures are from
http://www.ndt-ed.org/EducationResources/CommunityCollege/Ultrasonics/
Visit this web site for nice demonstrations.

Beamforming and steering

A specific spatial distribution of acoustic intensity (acoustic beam) can be generate by overlapping elementary point sources (interference).

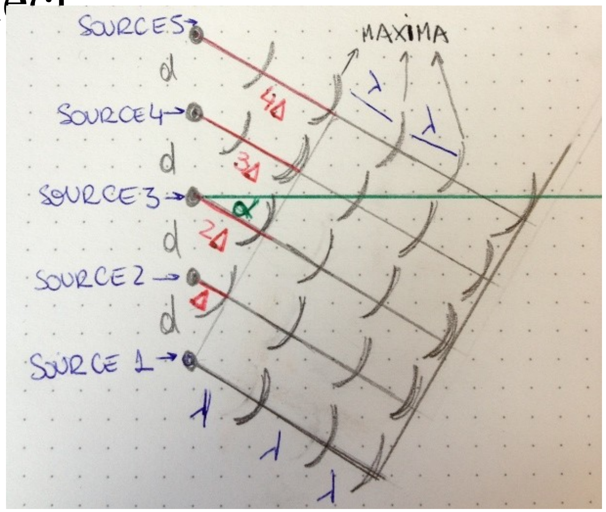


The Huygen-Fresnel principle: Wavefronts can be decomposed into a collection of point sources, each the origin of a spherical, expanding, wave.

Assignment

Assignment 9B: Simulate a phased array ultrasound wave

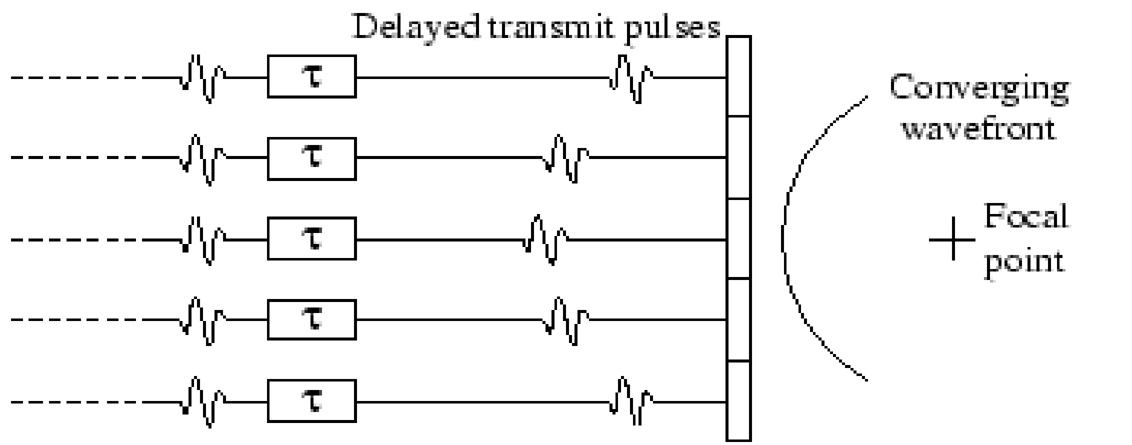
1. Simulate a wave pattern as generated by M radial point sources (see example in previous slide for M=5) on an image of dimensions 1m by 1m (with Nx,Ny pixels).
2. Alter wavelength, distance and number of points sources and/or aperture and determine what will give the narrowest possible "broadside" beam. All length measured in meters. Aperture is the distance of the two most distant sources (a=d*M).
3. Determine how to place the elements so as to generate a narrow beam with no side-lobes. Submit code with your final choice of M, aperture and wave lengths And once sentence each describing their effect.
4. Determine how to introduce phase delays to steer the beam in lateral directions. Your program should ask the user for the desired angle alpha. (Hint: The delay you have to compute corresponds to delta (red in the diagram here). There is a triangle that includes the desired angle alpha and delta. Use trigonometry to compute delta. Now, given the wavelength lambda, what phase does that delta correspond to? That is the phase you have to add to each source point so the beam has angle alpha.)

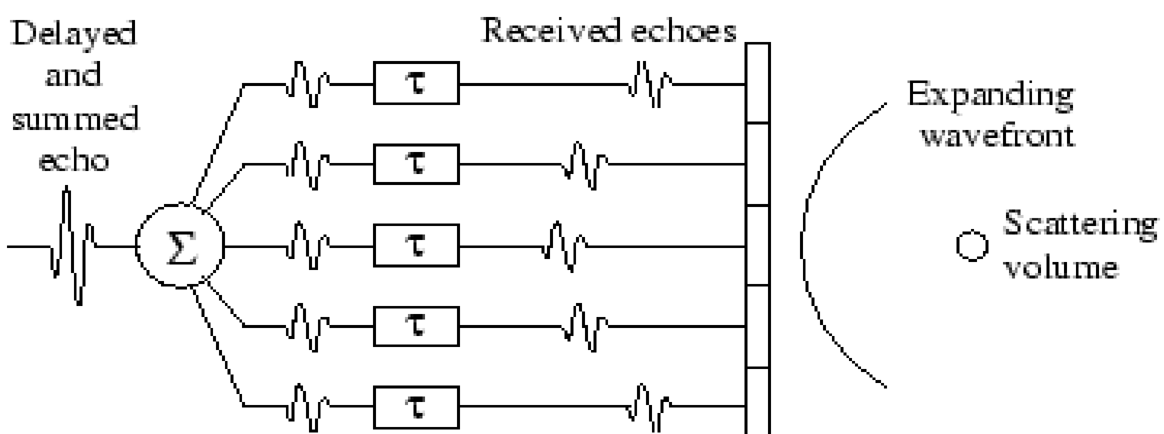


Submit code that makes on image for items 3 and 4.

Delay-sum beamforming

By properly delaying the sound pulse one can focus the bean to interfere constructively in a specific location

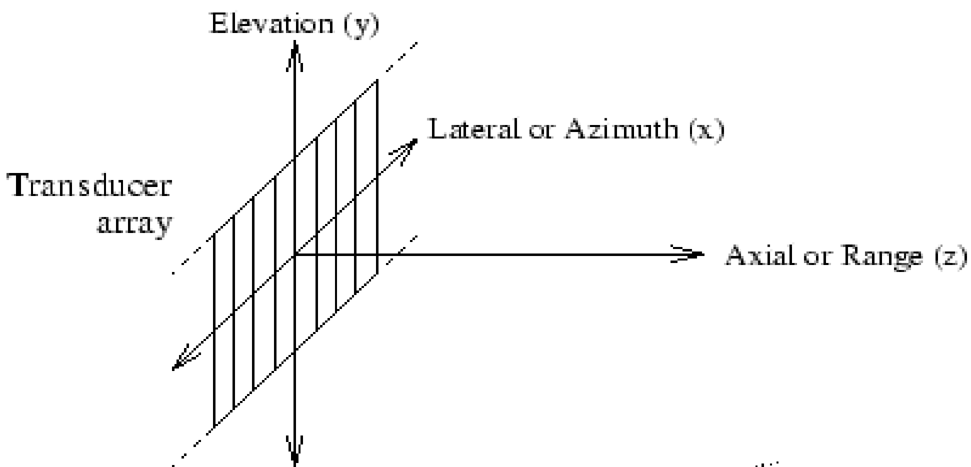


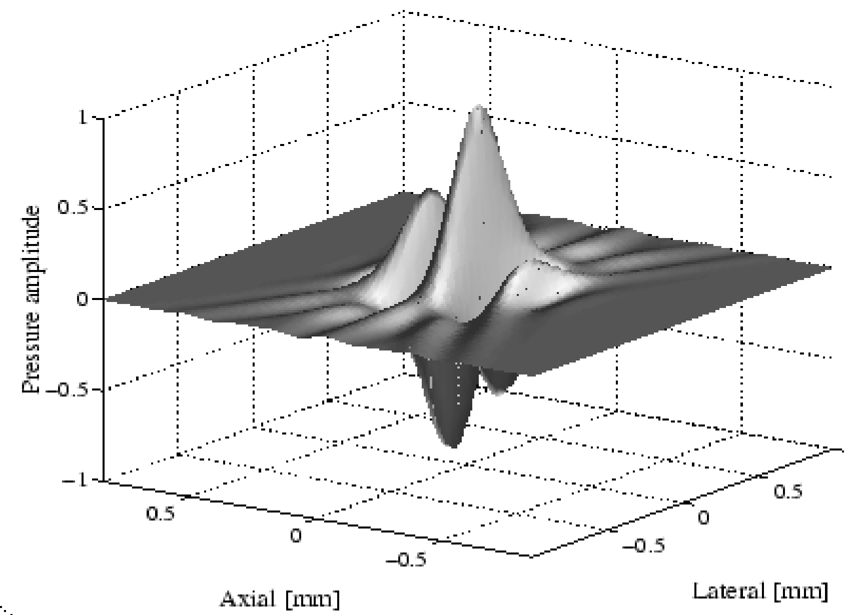


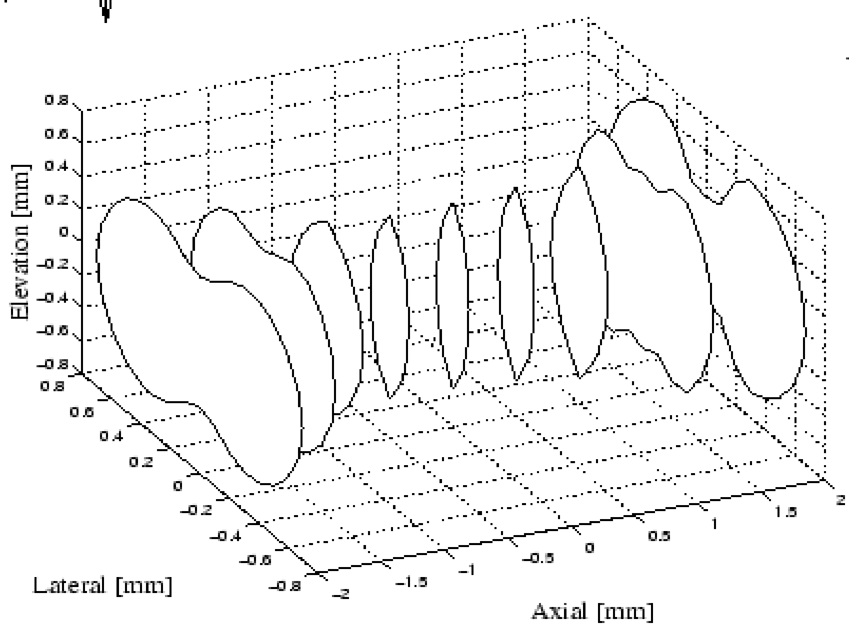
Figures are from http://dukemil.egr.duke.edu/Ultrasound/k-space/

Transducer array

Typical transducer array (phased array) generates a small focus and is steerable.







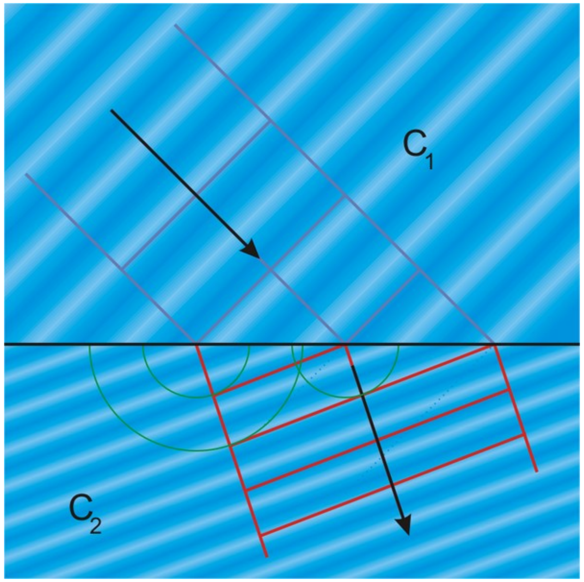
Reflection and scatter

Wave travels smoothly in an uniform medium (constant velocity).

Non-uniform medium (varying velocity) leads to

- Reflection (object larger than λ)
- Scatter (object smaller than λ)

Reflection and scatter can be understood using Huygens' Principle, e.g. Wave diffraction on a linear boundary (large relative to λ) :

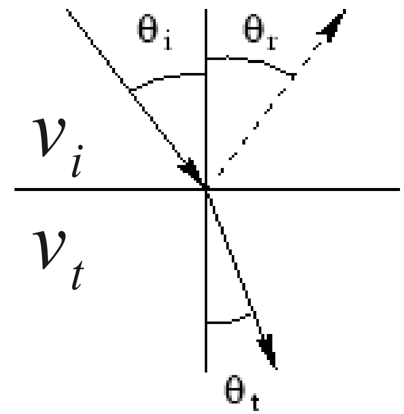


Scattering and reflection

The signal that returns from the tissue is a combination of multiple reflections and scatterers.

Reflection $d \gg \lambda$

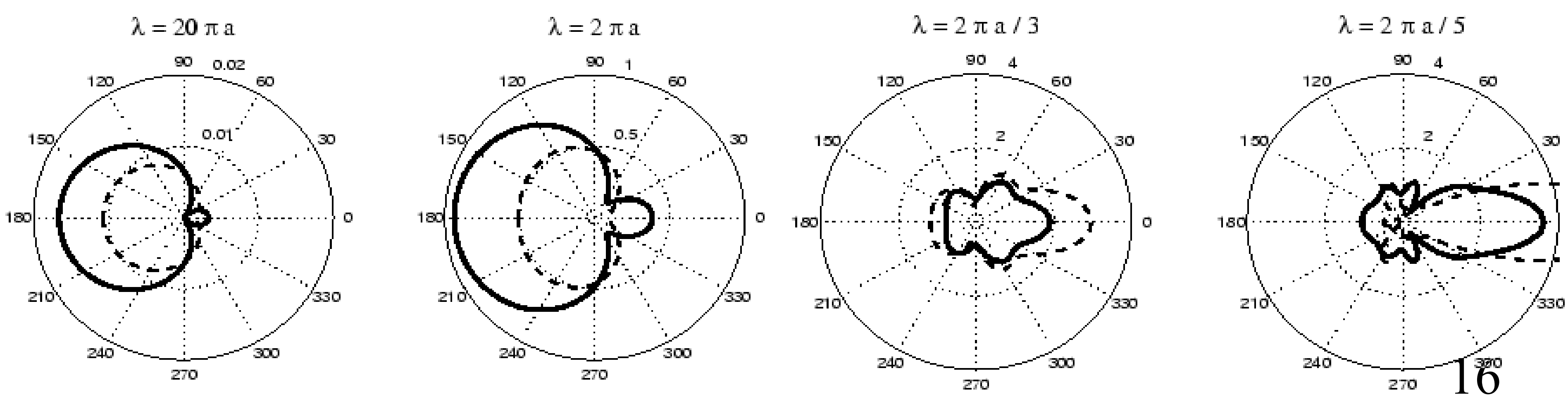
$$R = \frac{Z_t - Z_i}{Z_t + Z_i}$$



$$Z \propto \rho \frac{v}{\cos\theta}$$

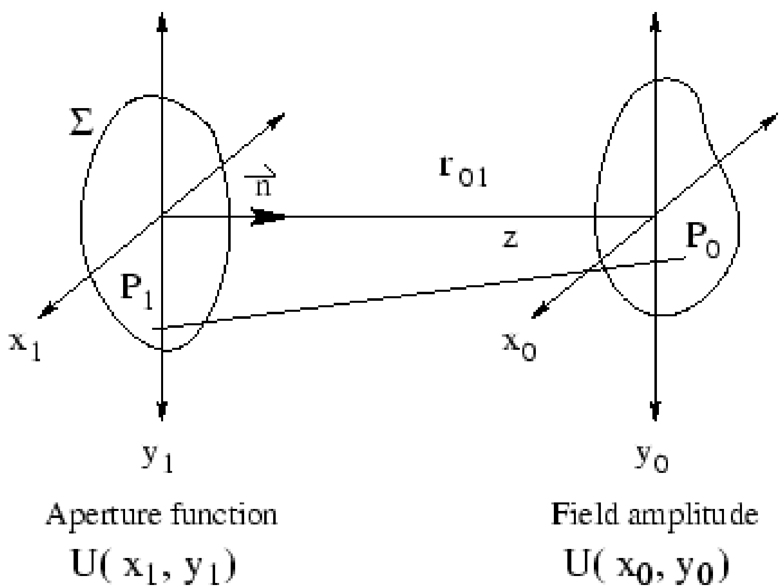
$$\frac{v_i}{v_t} = \frac{\sin\theta_i}{\sin\theta_t}$$

Scatter $d \approx \lambda$



Beam is determined by diffraction

The far-field beam pattern in transverse direction (x,y) can be calculated using Huygen's principle: Each point in the aperture is the source of a spherical wave



$$r_{01}=\sqrt{z^2+(x_1-x_0)^2+(y_1-y_0)^2}$$

$$P_1=(x_1, y_1) \qquad P_0=(x_0, y_0)$$

Spherical wave: $$h(P_0, P_1)=\frac{1}{j\lambda}\frac{\exp(jkr_{01})}{r_{01}}\cos(\boldsymbol{n}, \boldsymbol{r}_{01})$$

Superposition: $$U(P_0)=\int\int_\Sigma h(P_0, P_1)U(P_1)\,dP_1$$

Fraunhofer Approximation

After some approximations this simplifies to the Fourier transform

$$U(x_{0,}y_0) \propto \int \int U(x_{1,}y_1) e^{\frac{-i2\pi}{\lambda z}(x_0 x_1 + y_0 y_1)} dx_1 dy_1$$

Small angle approximation:

$$h(x_{0,}y_{0,}x_{1,}y_1) \approx \frac{1}{j\lambda z} \exp(jkr_{01})$$

Fresnel approximation:

$$r_{01} \approx z\left[1 + \frac{1}{2}\left(\frac{x_1 - x_0}{z}\right)^2 + \frac{1}{2}\left(\frac{y_1 - y_0}{z}\right)^2\right]$$

Fraunhofer approximation:

$$z \gg k(x_1^2 + y_1^2)$$

Diffraction limited resolution

One will not be able to resolve an object much smaller than the size of the beam focus. The aperture $U(x_1,y_1)$ and the beam $U(x_0,y_0)$ constitute a Fourier pair.

The uncertainly principle for this Fourier pair gives

$$\frac{\Delta x_0 \Delta x_1}{\lambda z} \geq \frac{1}{2}$$

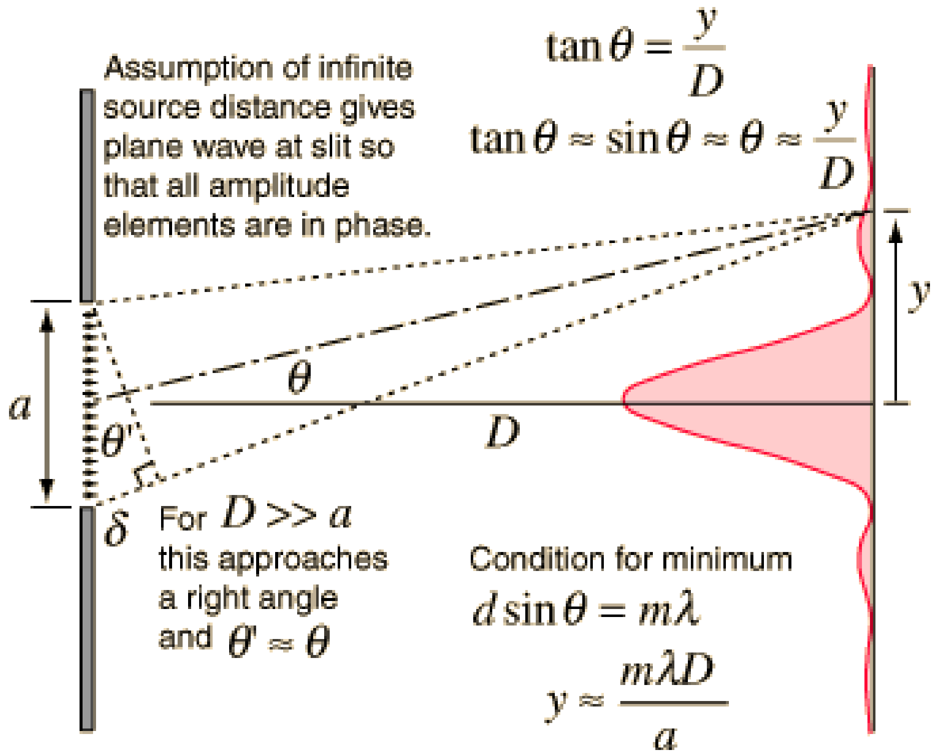
where $2\Delta x_0$ is width of the beam (resolution) and Δx_1 the size of the aperture.

$$2\Delta x_0 \geq \frac{\lambda z}{\Delta x_1}$$

Therefore good resolution is obtained with short wavelength, large aperture, and a close by object. This is true for any diffraction limited imaging system!

Rectangular aperture

A rectangular aperture $U(x_1,y_1)$ results in a beam $U(x_0,y_0)$ given by the corresponding sinc( ) function. This figure shows intensity $|U(x_0,y_0)|^2$



Sorry for changing notation here: $y = 2\Delta x_1, D = z, a = \Delta x_0$

Figure from http://hyperphysics.phy-astr.gsu.edu

Doppler Ultrasound

Blood cells in an artery are moving. When ultrasounds reflects on these moving objects the frequency of the reflected ultrasound shifts. This frequency shift if known as Doppler effect.

$$f' = \left(\frac{v}{v + v_r} \right) f$$

f' – frequency of reflected ultrasound
f – frequency of ultrasound sound
v – velocity of sound
v_r – velocity of reflecting object

Frequency shift is displayed as color.

Here blue is back-flow in carotid artery