



BME 50500: Image and Signal Processing in Biomedicine

Lecture 8: Medical Imaging Modalities MRI



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Medical Imaging

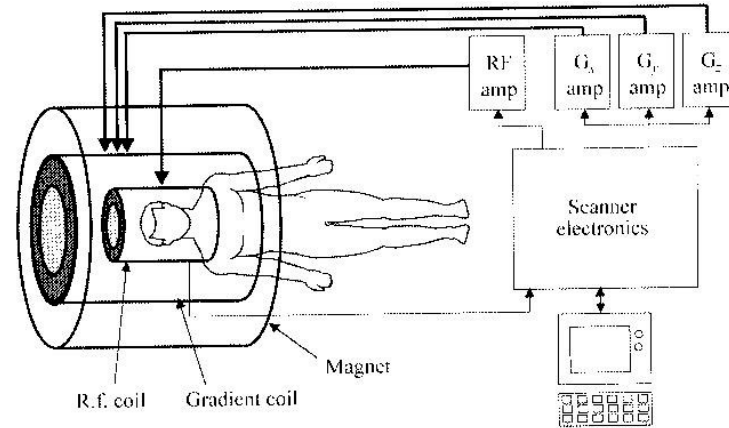
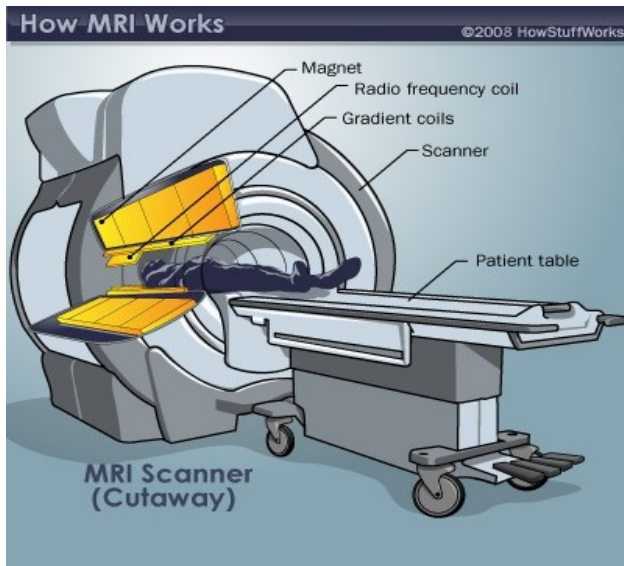
<i>Imaging Modality</i>	<i>Year</i>	<i>Inventor</i>	<i>Wavelength Energy</i>	<i>Physical principle</i>
X-Ray	1895	Röntgen (Nobel 1901)	3-100 keV	Measures variable tissue absorption of X-Rays
Single Photon Emission Comp. Tomography (SPECT)	1963	Kuhl, Edwards	150 keV	Radioactive decay. Measures variable concentration of radioactive agent.
Positron Emission Tomography (PET)	1953	Brownell, Sweet	150 keV	SPECT with improved SNR due to increased number of useful events.
Computed Axial Tomography (CAT or CT)	1972	Hounsfield, Cormack (Nobel 1979)	keV	Multiple axial X-Ray views to obtain 3D volume of absorption.
Magnetic Resonance Imaging (MRI)	1973	Lauterbur, Mansfield (Nobel 2003)	GHz	Space and tissue dependent resonance frequency of kern spin in variable magnetic field.
Ultrasound	1940-1955	many	MHz	Measures echo of sound at tissue boundaries.



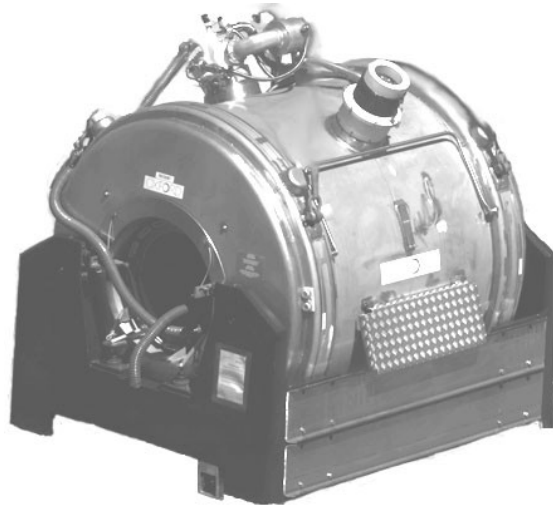
(Medical) imaging modalities overview

<i>Imaging Modality</i>	<i>What is being imaged</i>	<i>Resolution Scale</i>	<i>Limiting Factor for resolution</i>
Ultrasound	Sound reflection	1 mm	Wavelength aperture
PET	Isotope concentration, x-ray emission	0.5 cm	Light Intensity
X-ray	X-ray absorption	0.1 mm	Film resolution
CT	X-ray absorption	1 mm	Detector resolution
Millimeter wave imaging	reflection	1 mm	Detector resolution
Light Microscope	Light reflection	1 μ m	Numerical aperture
MRI	Proton density, Spin relaxation times T1 and T2 among other	1mm	Magnetic field strength (RF signal strength)

MRI - Equipment



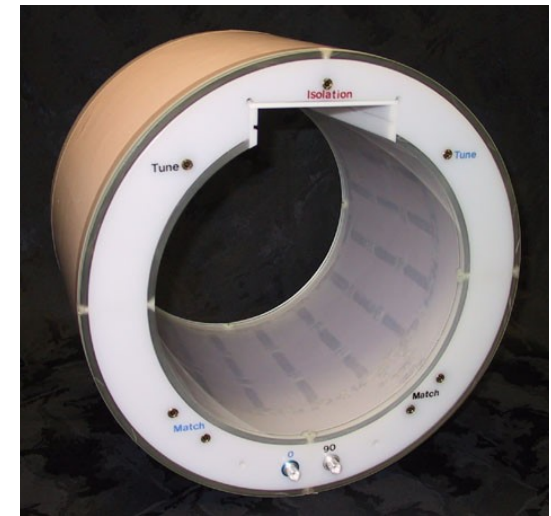
Magnet



Gradient Coil



RF Coil



Source: Joe Gati, photos



MRI – Basic Recipe

➔ 1) Put subject in big magnetic field

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2) Transmit radio waves into subject [about 3 ms]

Exposure to radio frequency magnetic field will synchronize this precession.

3) Turn off radio wave transmitter

The coherent precession continues but decays slowly due to interactions with magnetic moments of surrounding atoms and molecules (tissue dependent!)

4) Receive radio waves re-transmitted by subject [10-110ms]

The coherent precession (oscillation) generates a current in an inductive coil. The detected signal is called magnetic nuclear resonance.

5) Store measured radio wave data vs. time

Now go back to 2) to get some more data with different magnetic fields and radio frequencies. (here lies the Art of MRI!)

6) Process raw data to reconstruct images



MRI – Big Magnet

Very strong

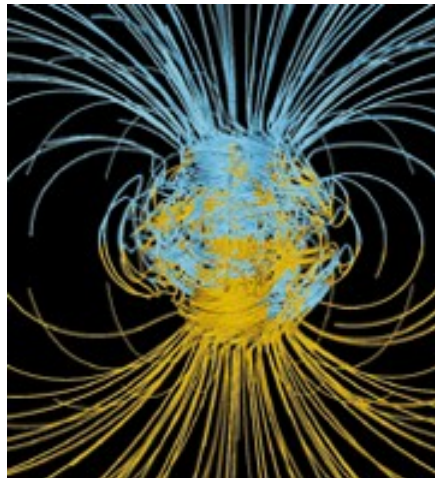
1 Tesla (T) = 10,000 Gauss

Earth's magnetic field = 0.5 Gauss

4 Tesla = $4 \times 10,000 \div 0.5 = 80,000X$ Earth's magnetic field

Continuously on

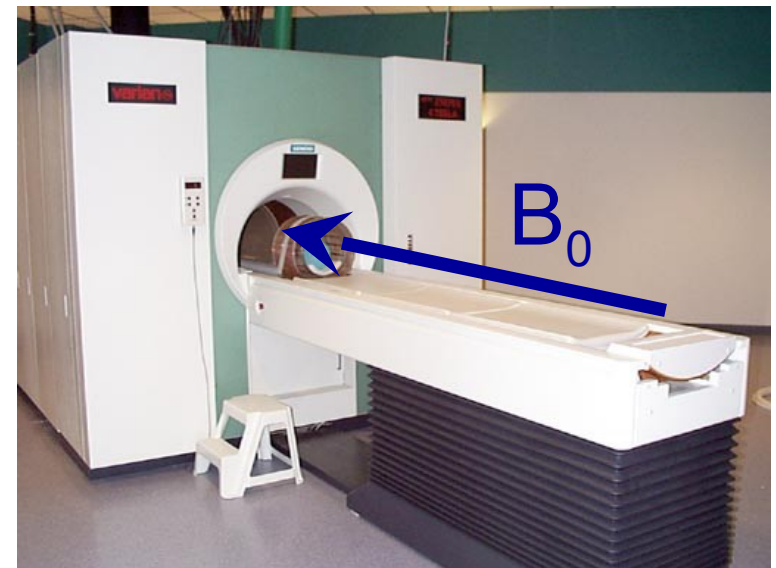
Main field = B_0



Source: www.spacedaily.com

$\times 80,000 =$

Robarts Research Institute 4T

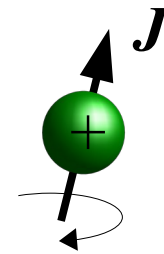




MRI – Nuclear Spin

Nucleus has a quantum mechanical property called “spin” quantized by I . ($I=1/2$ for a proton in H_2O). Spin can be thought of as a spinning mass with an angular momentum J .

$$|J| = \frac{h}{2\pi} \sqrt{I^2 + I}$$



Since the particle is electrically charged this spinning will generate a magnetic moment μ :

$$\mu = \gamma J$$

The gyromagnetic ratio γ is specific to each nucleus.

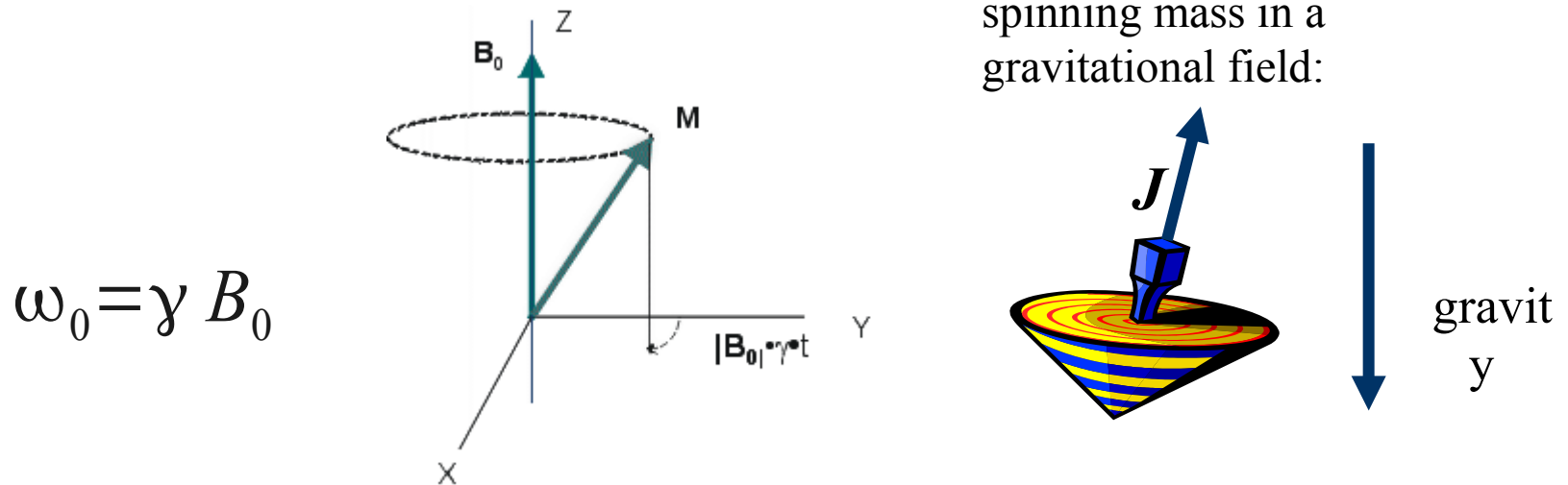
As we will see the magnetic fields and radio frequency (RF) are tuned to a specific value of γ , i.e. to a specific nucleus.



MRI – Nuclear Spin in Magnetic Field

When a spin is placed in a homogeneous external magnetic field B_0 it precesses at Larmor frequency ω_0 .

The effect is analogous to a spinning mass in a gravitational field:



Quantum mechanics however dictates that the values for the z-orientation of J (and μ) can only be:

$$\mu_z = \gamma J_z = \frac{\gamma h}{2\pi} m_I$$

with $m = \pm 1/2$ for $I = 1/2$.



MRI – Nuclear Spin

Properties on nuclei found at high abundance in the body:

Nucleus	Atomic Number	Atomic Mass		$\gamma/2\pi$ (MHz/T)	MRI Signal
Proton, ^1H	1	1	$\frac{1}{2}$	42.58	yes
Phosphorus, ^{31}P	15	31	$\frac{1}{2}$	17.24	yes
Carbon, ^{12}C	6	12	0		no
Oxygen, ^{16}O	8	16	0		no
Sodium, ^{23}Na	11	23	$\frac{3}{2}$	11.26	yes

MRI can be performed with odd odd atomic mass (non-zero spin)



Most frequent medical imaging is performed with ^1H (proton)

abundant: high concentration in human body

high sensitivity: yields large signals

1.5T magnet uses RF at 3.87 MHz for proton imaging.



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6) Process raw data to reconstruct images

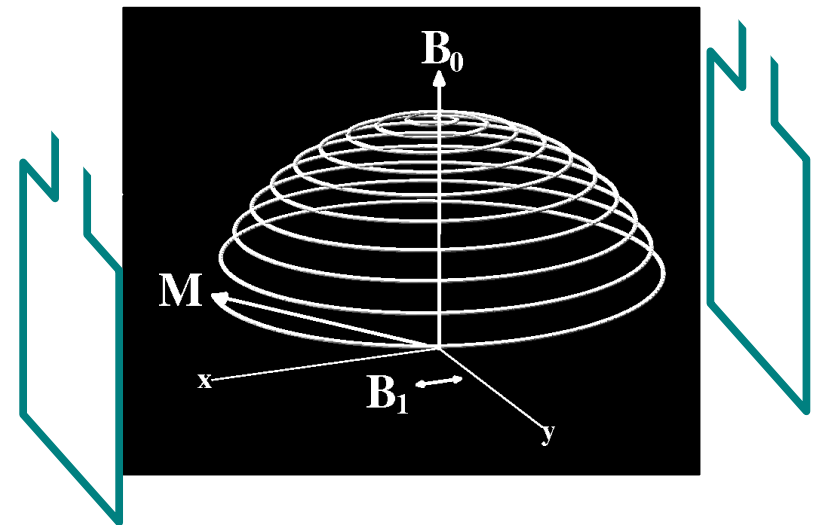


MRI – RF pulse

If we apply in addition to B_0 a field component B_1 ($\ll B_0$) in the x -direction *oscillating at frequency* ω_0 the trajectory for M will be:

$$B_x(t) = B_1 \sin(\omega_0 t)$$

emitting
RF coil

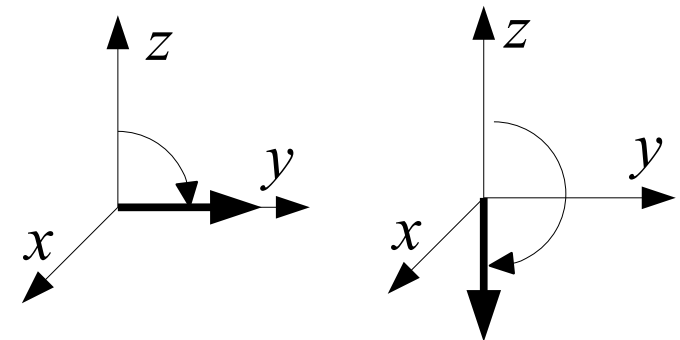


This time varying B_1 field is applied for a short time (few ms) with an RF coil at the x -axis. The final “flip” angle depends on the length of this RF pulse and the strength of B_1 .

Useful flip angles are:

$\alpha = 90^\circ$ M_z is converted into M_y

$\alpha = 180^\circ$ M_z is converted into $-M_z$





MRI – RF pulse

The Swing Analogy:

Oscillating spins generate bulk magnetization M_z lined up with B_0 :

A bunch of kids are swinging at different swings, all with the same frequency but out of phase. The average weight of the kids is straight down from the pole – it is “aligned” with external gravity.

RF pulse (oscillating B_1) generates transverse M_x , M_y oscillation:

If parents push a little bit on every swing, in synchrony, and at the natural frequency of the swings, soon all kids are swinging together in phase. The average weight of the kids is now oscillating back and forth, i.e. there is now a oscillating transverse component.

How well they are lined up at the end depends on how often and how strong they were pushed.

Note that if the parents pushed at a frequency other than the natural frequency of the swings their effort would not amount to much.



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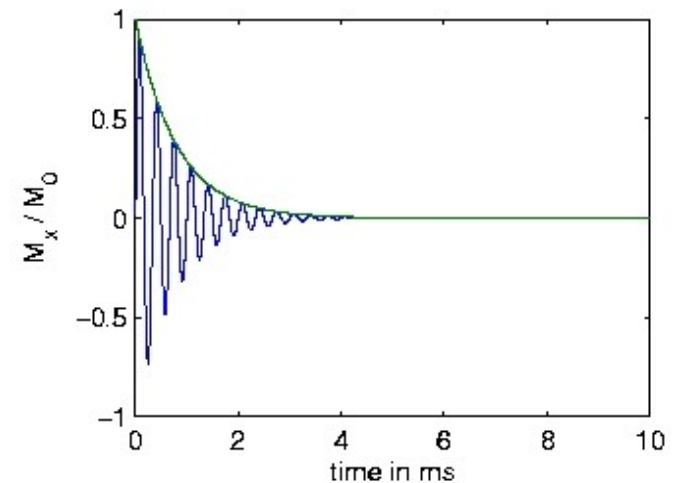
MRI – Free Precession - T_2 decay

After the RF pulse the system is left only with B_0 . Any contribution in the transverse direction will precess around B_0 at ω_0 . Lets now consider the second term:

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \mathbf{B} - \frac{1}{T_2} \begin{bmatrix} M_x \\ M_y \\ 0 \end{bmatrix} - \frac{1}{T_1} \begin{bmatrix} 0 \\ 0 \\ M_z - M_0 \end{bmatrix}$$

This term indicates that M_x, M_y will decay exponentially with a time constant T_2 . Together with the precession this gives a damped oscillation, e.g. after a 90° pulse:

$$\begin{bmatrix} M_x \\ M_y \end{bmatrix} (t) = M_0 e^{-\frac{t}{T_2}} \begin{bmatrix} \sin(-\omega_0 t) \\ \cos(-\omega_0 t) \end{bmatrix}$$





MRI – Free Precession - T_2 decay

The reason for this decay process is that each spins each see a slightly different local field around them. Each then oscillates at a slightly different frequency. The spins will be therefore quickly out of step, and the bulk transverse magnetization will disappear.

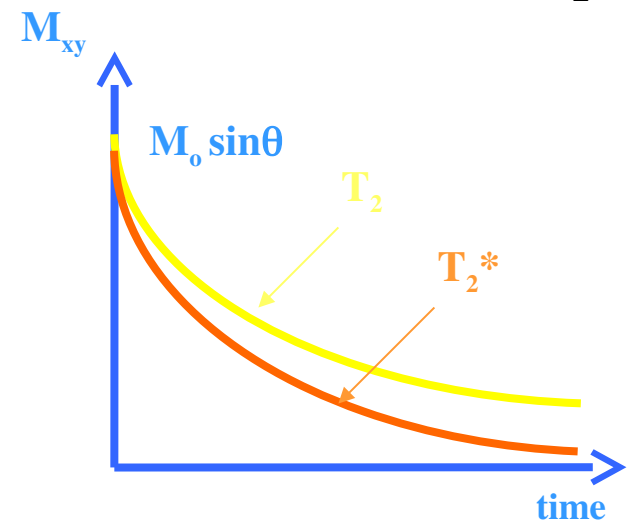
The local magnetic fields are not the same because:

1. Each spin sees the magnetic field generated by other spins in the molecule. Quantified with T_2 . (“spin-spin relaxation”)
2. The field B_0 is not perfectly homogeneous. Quantified with T_2^+ and about 100 shorter than T_2 .

Total effect is T_2^* :

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2^+}$$

T_2^* dominated by T_2^+ and is just a few ms.





MRI – Free Precession - T_1 relaxation

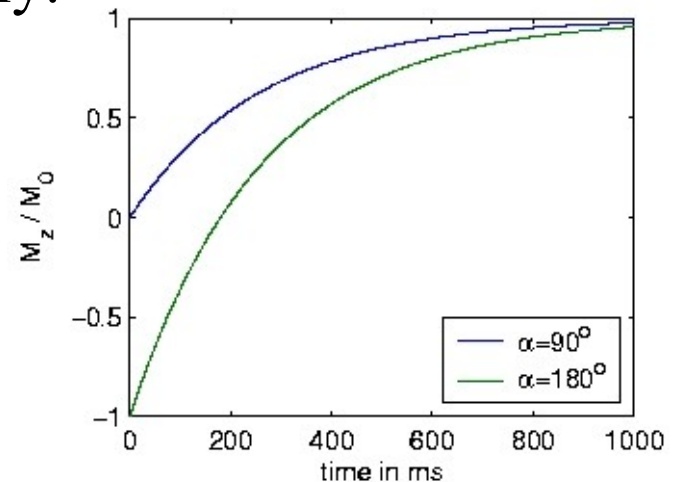
The third term in the Bloch equation describes the relaxation of the longitudinal magnetization M_z :

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \mathbf{B} - \frac{1}{T_2} \begin{bmatrix} M_x \\ M_y \\ 0 \end{bmatrix} - \frac{1}{T_1} \begin{bmatrix} 0 \\ 0 \\ M_z - M_0 \end{bmatrix}$$

This is an exponential relaxation back to the equilibrium value M_0 , e.g. after a 90° pulse and a 180° respectively:

$$M_z(t) = M_0 \left(1 - e^{-\frac{t}{T_1}} \right)$$

$$M_z(t) = M_0 \left(2 - e^{-\frac{t}{T_1}} \right)$$



This exponential recovery represents the return of the system to its equilibrium condition $M_z = M_0$, whereby the spins lose energy to the surrounding lattice (“spin-lattice relaxation”)



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MRI – RF pulse

Now an oscillating B_1 field perpendicular to B_0 will be applied at resonant (precession) frequency ω_0

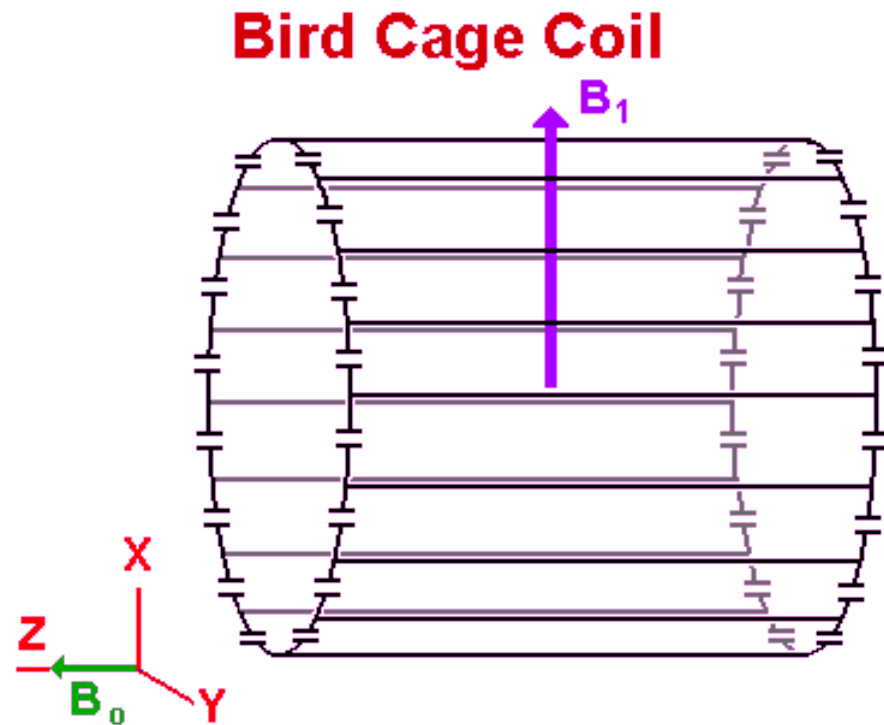
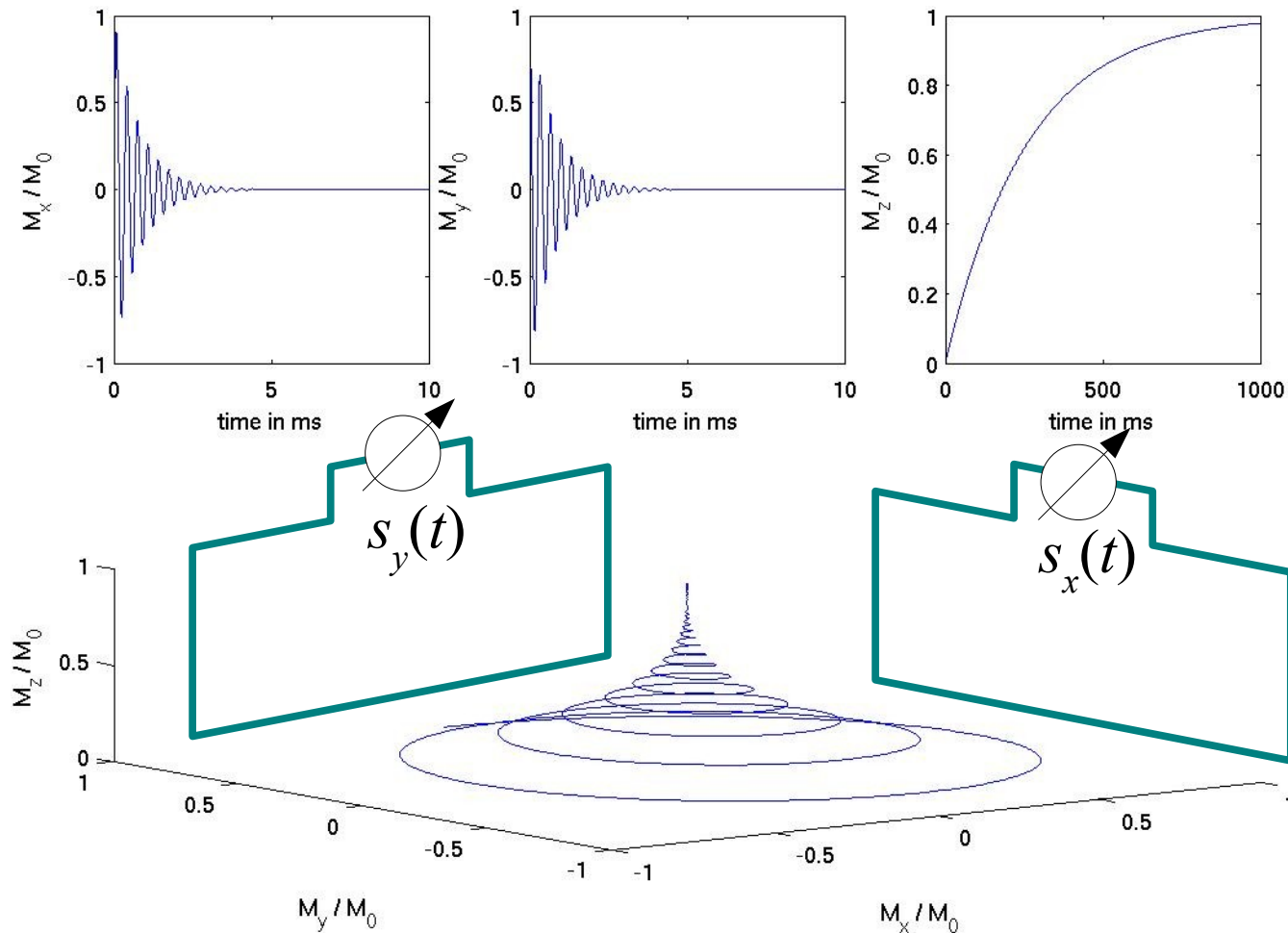


Image courtesy: Jeff Hornak

MRI – Free Precession

The overall free precession of the bulk magnetization \mathbf{M} after RF pulse of $\alpha=90^\circ$ is then

Free precession after $\alpha = 90^\circ$ RF pulse



receiving
RF coil



MRI – Free Induction Decay

This precessing magnetization can be measured inductively with an receiver coil tuned to the resonant frequency ($\omega_0=3.87$ MHz for ^1H). The detected signal is called the **Free Induction Decay** (FID). If we detect it in with a coil in x and y axis we can construct a complex variable

$$s(t) = s_x(t) + i s_y(t) \propto M_x(t) + i M_y(t) = M_{xy}(0) e^{-t/T_2^*} e^{-i\omega_0 t}$$

$M_{xy}(0)$ denotes here the magnitude of the M_x, M_y at the end of the RF pulse, i.e. at $t=0$ of the free precession. Its value is dependent of the specific pulse sequence and is affected typically by the decay times T_1 and T_2 .

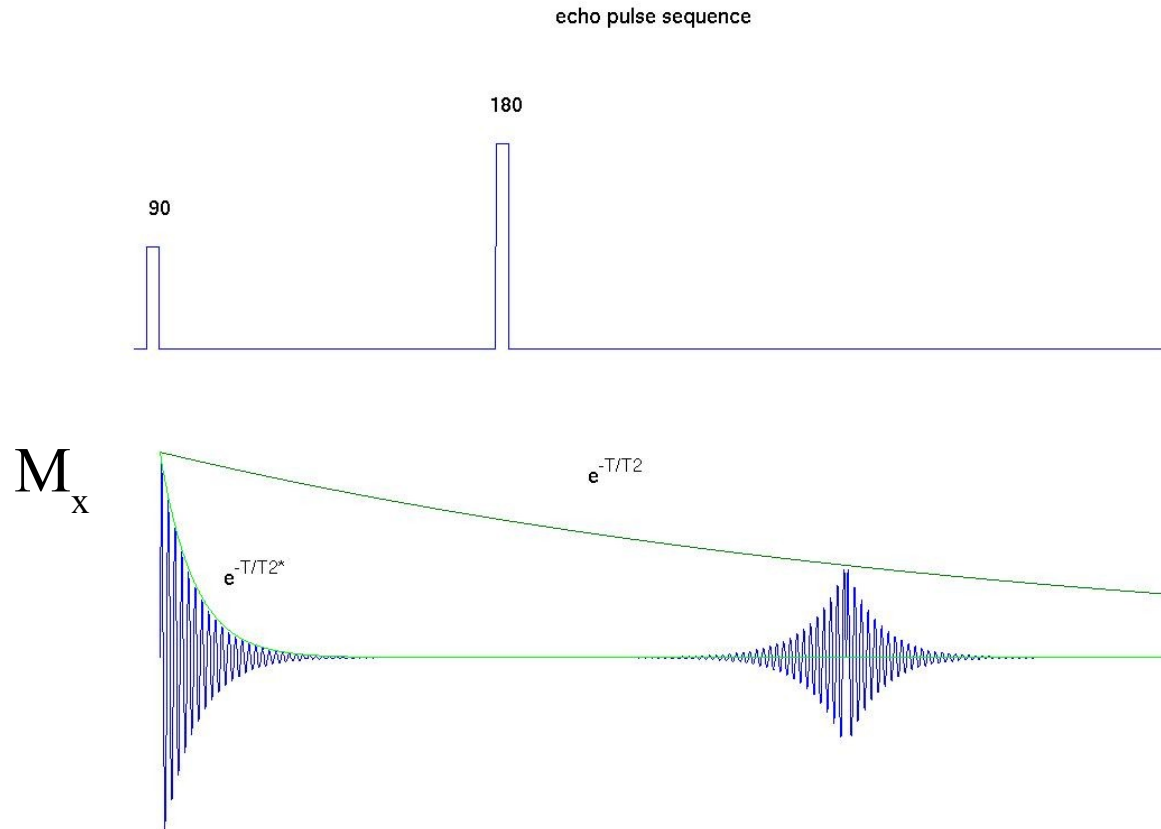
By modifying the RF pulses and measuring the magnitude of $s(t)$ one can make estimate the decay times T_1 and T_2 .



MRI – Pulse sequences to estimate T1, T2

T2 – Echo pulse sequence

$90^\circ - \tau - 180^\circ$: the detected signal magnitude is $\propto \exp\left(-\frac{\tau}{T_2}\right)$



Assignment 8: Generate graphics representing the pulse sequence and FID for inversion recovery and echo pulse.



MRI – Nuclear Magnetic Resonance (NMR)

The decay constants T_1 and T_2 depend on physical properties of the resonating sample. By measuring the decay constants one can therefore deduce what is in the sample.

In the 70' it was realized that this may be used for medical applications (Damadian)

Tissue	T1 (ms)	T2 (ms)
Fat	260	80
Muscle	870	45
Brain (gray matter)	900	100
Brain (white matter)	780	90
Liver	500	40
Cerebrospinal fluid	2400	160



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6) Process raw data to reconstruct images



MRI – How to generate images using NMR

Nuclear spins resonate at a frequency proportional to the external magnetic field

$$\omega = \gamma B_0$$

Basic idea of MRI: Change the B_0 field with space and the resonance frequency will change with space.

$$\omega(\mathbf{r}) = \gamma B_0(\mathbf{r})$$

The detected resonance signal (FID) contains multiple frequency components each giving information about a different portion of space!



MRI – Signal detected in MRI

Recall that the signal due to the bulk magnetization precessing at ω detected in the x and y coils can be written as:

$$s(t) = s_x(t) + i s_y(t) \propto M_{xy}(0) e^{-t/T_2^*} e^{-i\omega t}$$

Signal intensity scales with $M_{xy}(0)$ - the magnitude of the transverse magnetization at the end of the RF pulse. $M_{xy}(0)$ is proportional to the number of resonating spins in the material, or the proton density $\rho(\mathbf{r})$. It is dependent on the tissue and therefore dependent on space \mathbf{r} .

MRI generates images of $\rho(\mathbf{r})$!

$M_{xy}(0)$ also depends on the specifics of the pulse sequence. By manipulating the pulse sequence MRI can generate images of $\rho(\mathbf{r})$ that are modulated by physical properties that affect T_1 or T_2 .



MRI – Signal detected in MRI

The main idea is to apply a B_0 field with a magnitude that also depends on space, so that the frequency of the resonance signal relates to space, $\omega(\mathbf{r}) = \gamma B_0(\mathbf{r})$:

$$s(t) \propto e^{-t/T_2^*} \rho(\mathbf{r}) e^{-i\gamma B_0(\mathbf{r})t}$$

(where we have ignored the effect of T_1 and T_2). The signal emitted by the entire body is then the sum over space:

$$s(t) \propto e^{-t/T_2^*} \int_{body} d\mathbf{r} \rho(\mathbf{r}) e^{-i\gamma B_0(\mathbf{r})t}$$

Note that $B_0(\mathbf{r})$ is parallel to the z -axis, only its magnitude may now depend on the location in space \mathbf{r} .



MRI – Signal detected in MRI

For reconstruction it will be useful to define new signal that is 'demodulated' and without the T_2^* decay:

$$S(t) = s(t) e^{t/T_2^*} e^{i\omega_0 t}$$

Define also $\Delta B_z(\mathbf{r})$ as the difference of $B_0(\mathbf{r})$ over main B_0 :

$$\Delta B_z(\mathbf{r}) = B_0(\mathbf{r}) - \omega_0 / \gamma$$

With this the MRI *imaging equations* becomes

$$S(t) = \int_{body} d\mathbf{r} \rho(\mathbf{r}) e^{-i\gamma \Delta B(\mathbf{r}) t}$$



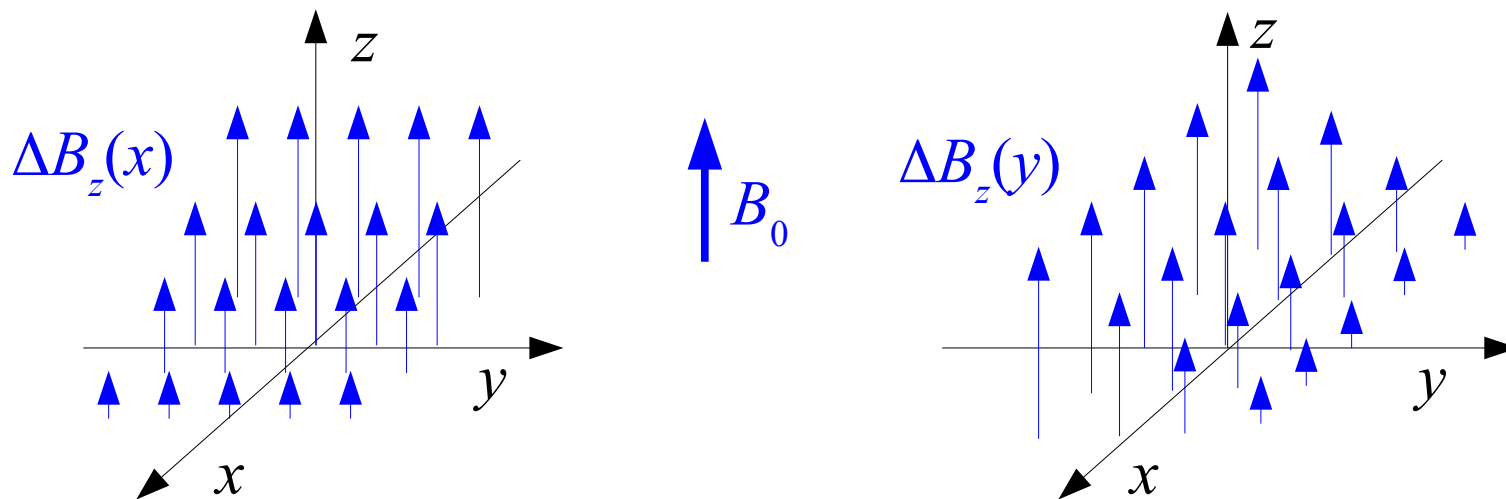
MRI – B_0 gradient, frequency encoding

Lets assume we need spatial resolution in only one direction. For instance x . So we want to recover (ignoring z direction for now):

$$g(x) = \int dy \rho(x, y)$$

To do so, we apply a contribution B_0 that changes linearly with x . The strengths of these 'x-gradient' is given by the constants G_x .

$$\Delta B_z(\mathbf{r}) = G_x x$$





MRI – B_0 gradient, frequency encoding

The imaging equation is now

$$S(t) = \int dx g(x) e^{-i\gamma G_x x t}$$

To put this in a more familiar notation lets define a new variable

$$k_x = \gamma G_x t \quad \gamma = \gamma / 2\pi$$

$$S(k_x) = \int dx g(x) e^{-i2\pi k_x x}$$

Evidently the detected signal $S(k)$ is a Fourier transform of $g(x)$, and we can recover it with the inverse Fourier transform.

$$g(x) = \int dk_x S(k_x) e^{i2\pi k_x x}$$

This method is therefore called *frequency encoding*. Obviously we can also apply a G_y gradient and obtain $g(y)$.



MRI – Axial Reconstruction

By combining x, y gradients linearly we can get gradients that at an arbitrary orientation ϕ :

$$\Delta B_z(\mathbf{r}) = G_x x + G_y y = \mathbf{G}_\phi \cdot \mathbf{r}$$

$$\mathbf{G}_\phi = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = G_\phi \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} \quad \mathbf{r} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The signal we obtain is then a Fourier transform of $\rho(\mathbf{r})$ along that direction (the orthogonal directions are summed).

$$\mathbf{k}_\phi = \begin{bmatrix} k_x \\ k_y \end{bmatrix} = k \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}$$

$$k = \gamma G_\phi t$$

$$S(t, \phi) = \int d\mathbf{r} \rho(\mathbf{r}) e^{-i\gamma \mathbf{G}_\phi \cdot \mathbf{r} t}$$

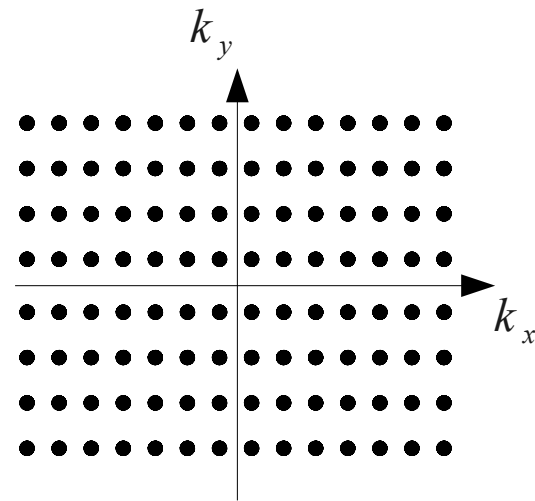
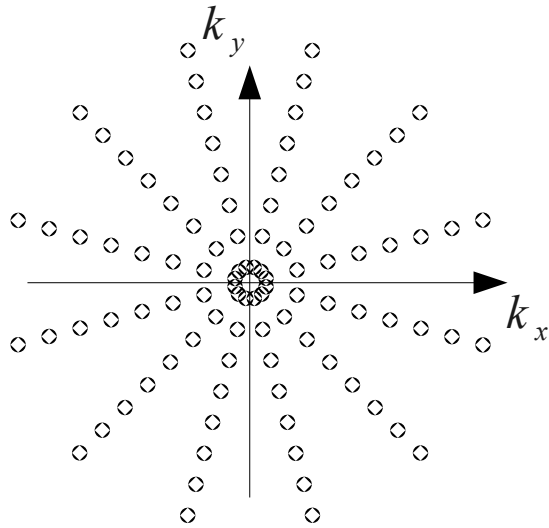
$$\mathbf{k}_\phi = \gamma \mathbf{G}_\phi t$$

$$S(k, \phi) = \int d\mathbf{r} \rho(\mathbf{r}) e^{-i2\pi \mathbf{k}_\phi \cdot \mathbf{r}}$$



MRI – k-space

Signals taken at multiple angles ϕ cover the k-space and allow therefore reconstruction (left).

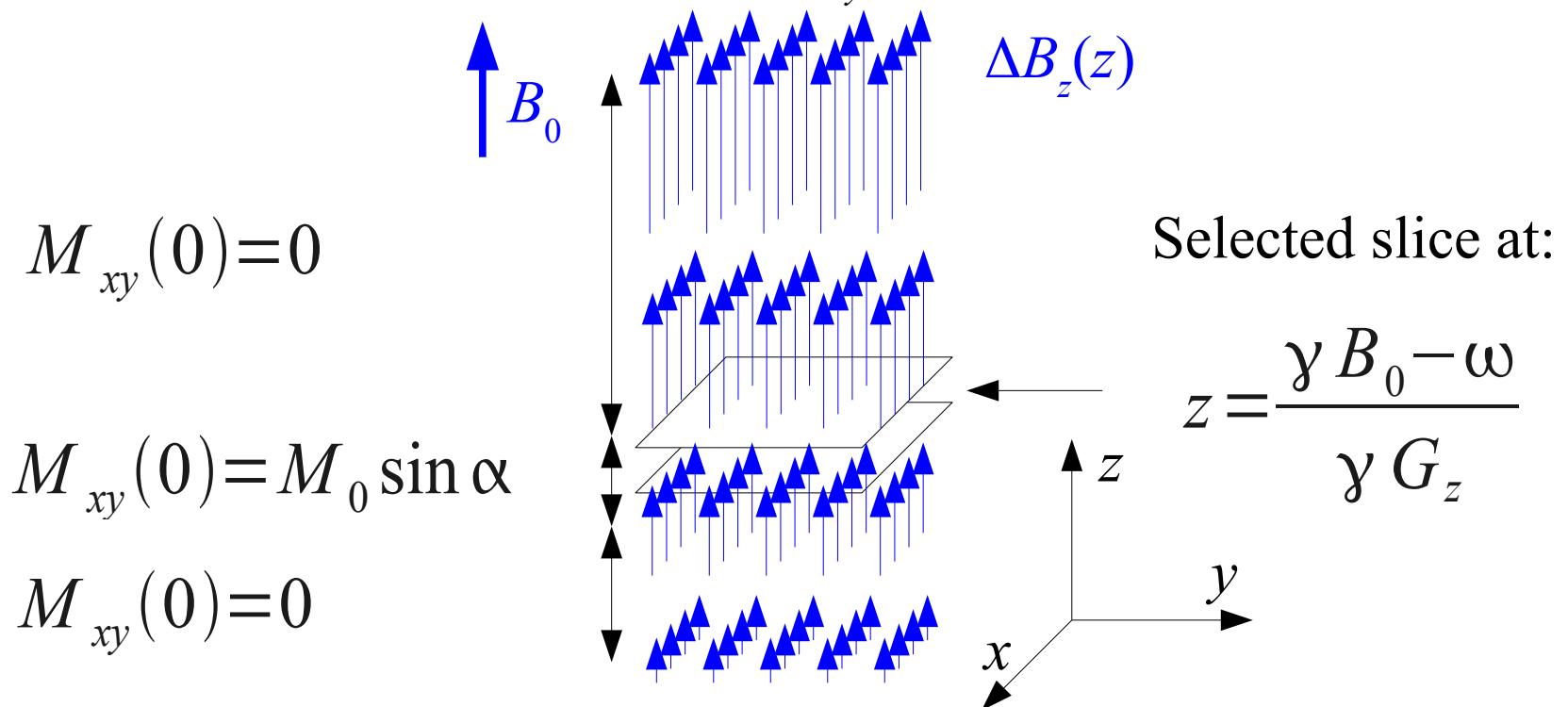


Is there a pulse sequence that can sample the Fourier space evenly as shown on the right so that we can use direct 2D Fourier inverse?



MRI – Slice selection

So far we considered gradients applied *after* the RF pulse during free precession. A gradient G_z *during* the RF pulse will select a transversal slice that satisfies the *resonance condition*: The RF pulse affects the spin precession coherently only if the frequency matches the B_z field. For the rest $M_{xy} = 0$ after α pulse.

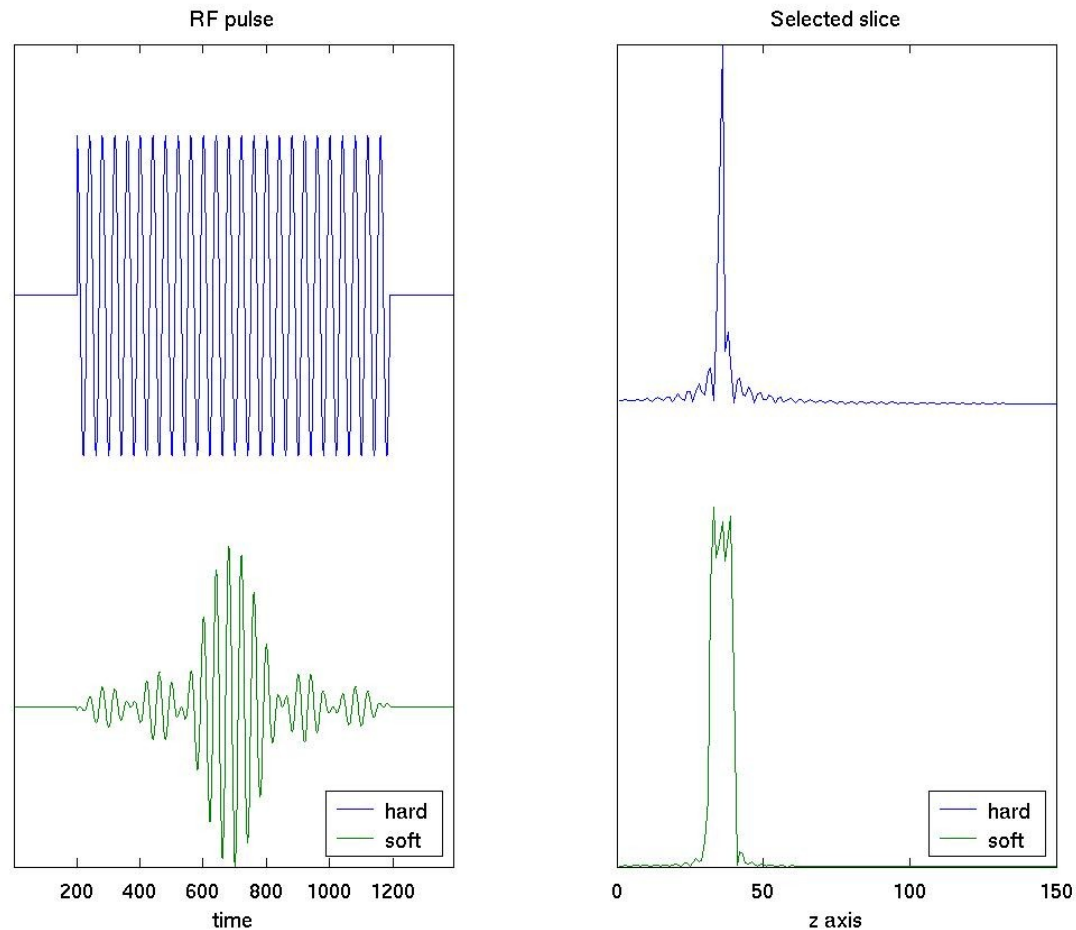


Only this slice will generate a signal!



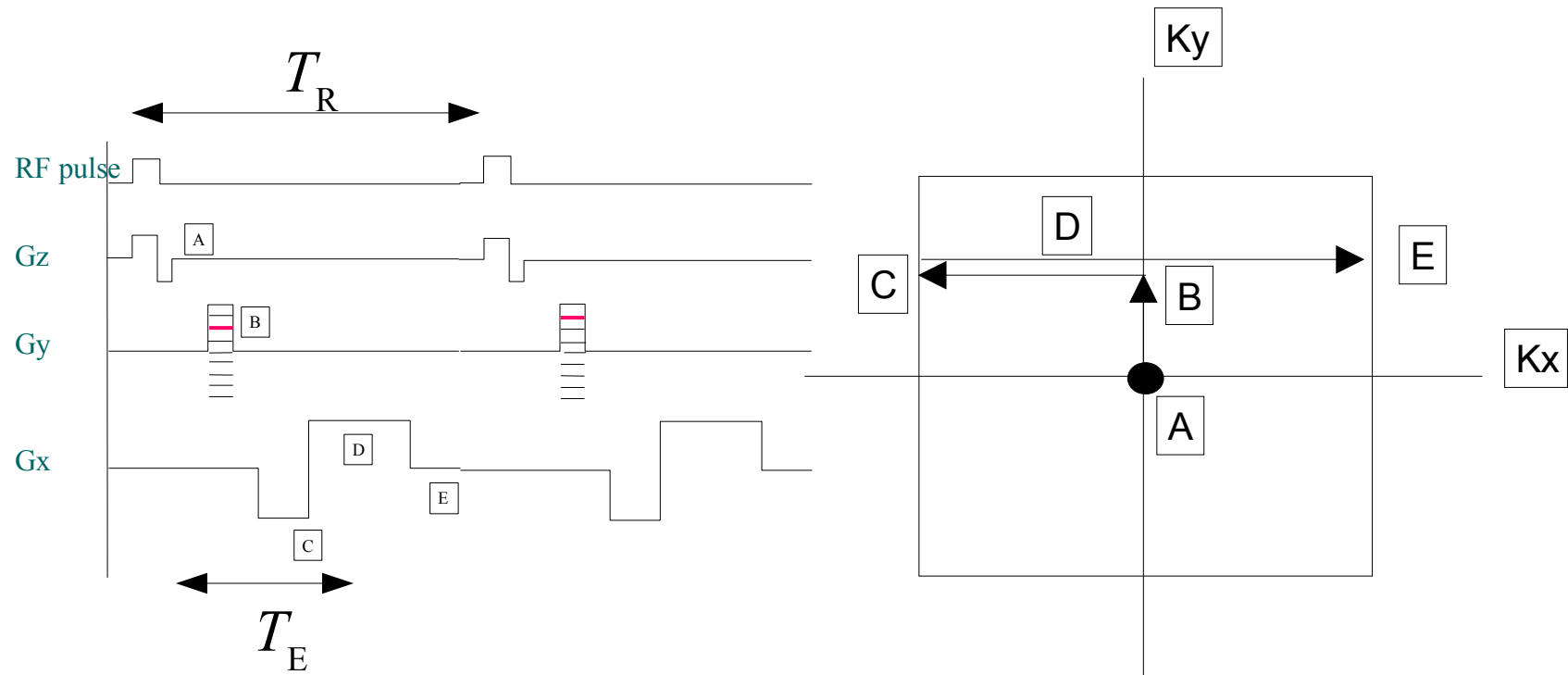
MRI – Slice selection

Note that a “hard” RF pulse contains high frequency components. It is therefore less selective in space as a “soft” pulse (sinusoid modulated by a sinc function) - $\sin(\omega_0 t) * \text{sinc}(\omega t)$:

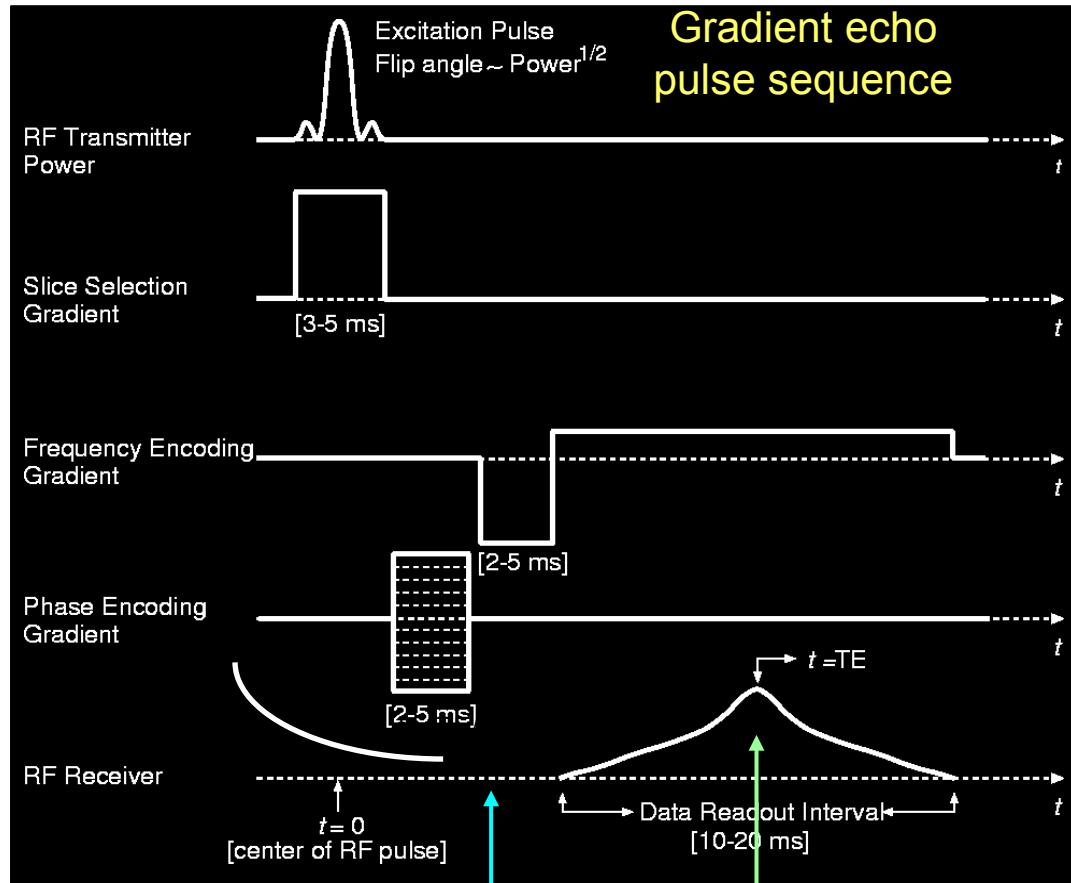


MRI – a pulse sequence example

Example for a full pulse sequence with gradient echo and the corresponding path in k-space:



MRI – a pulse sequence example



Gradient echo pulse sequence

Echos – refocussing of signal

Spin echo:

use a 180 degree pulse to “mirror image” the spins in the transverse plane

when “fast” regions get ahead in phase, make them go to the back and catch up

- measure T2
- ideally TE = average T2

Gradient echo:

flip the gradient from negative to positive
make “fast” regions become “slow” and vice-versa

- measure T2*
- ideally TE ~ average T2*

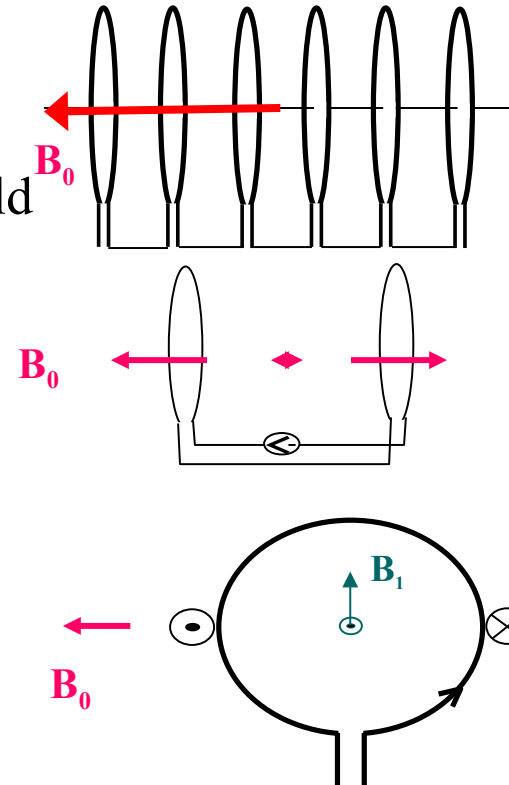
A gradient reversal (shown) or 180 pulse (not shown) at this point will lead to a recovery of transverse magnetization

TE = time to wait to measure refocussed spins



MRI – Summary for Magnetic fields

- Main Magnet
 - High, constant, Uniform Field, B_0 .
 - Causes a bulk magnetization of the body (“magnetizes”).
- Gradient Coils
 - Produce linear gradients in this field (B field changing in space).
 - Purpose is to resolve RF signal in space.
 - G_x , G_y , & G_z
- RF coils
 - Transmit: B_1 Excites NMR signal (FID).
 - Synchronizes spin precession
 - Receives RF signal from the body.





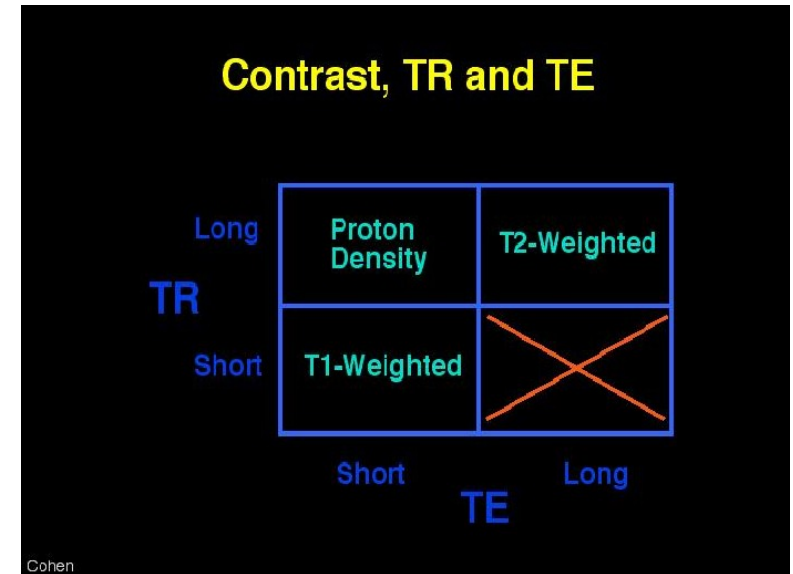
MRI – Summary of issues affecting resolution

- Signal strength is proportional to number of protons and B_0 field magnitude.
- Making large B_0 field requires very large currents are needed (using superconducting coils) which is expensive.
- RF signals emitted by the body is generally small.
- To have good signal to noise ratio you can make larger voxels (more protons) or stronger B_0 field (spend more money)
- Therefore, good resolution (small voxels) require large B_0 field (expensive scanner)

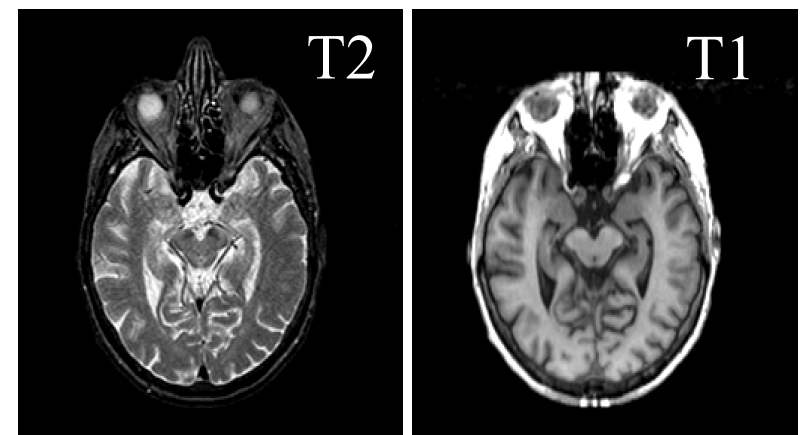


MRI – Contrast properties

- The strength of the NMR signal produced by precessing protons in a tissue depends on
 - T1, T2 of the tissue.
 - The density of protons in the tissue.
 - Motion of the protons (flow or diffusion).
 - The MRI pulse sequence used
- In a T1 “weighted” image the pulse sequence is chosen so that T1 has a larger effect than T2.
- Images can also be made to be T1, T2 proton density or flow/diffusion weighted.



Source: Mark Cohen





MRI – Contrast, T1, T2

- MRI Contrast is created since different tissues have different T1 and T2.
- Gray Matter: (ms) T1= 810, T2= 101
- White Matter: (ms) T1= 680, T2= 92
- Bone and air are invisible.
- Fat and marrow are bright.
- CSF and muscle are dark.
- Blood vessels are bright.
- Gray matter is darker than white matter.

