BME I5000: Biomedical Imaging

Lecture 2
X-Ray Imaging

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some slides inspired by lecture notes of Andreas H. Hilscher at Columbia University.

Schedule

1. Introduction, Spatial Resolution, Intensity Resolution, Noise
2. X-Ray Imaging, Mammography, Angiography, Fluoroscopy
3. Intensity manipulations: Contrast Enhancement, Histogram Equalization
4. Computed Tomography
5. Image Reconstruction, Radon Transform, Filtered Back Projection
6. Positron Emission Tomography
7. Maximum Likelihood Reconstruction
8. Magnetic Resonance Imaging
9. Fourier reconstruction, k-space, frequency and phase encoding
10. Optical imaging, Fluorescence, Microscopy, Confocal Imaging
11. Enhancement: Point Spread Function, Filtering, Sharpening, Wiener filter
12. Segmentation: Thresholding, Matched filter, Morphological operations
13. Pattern Recognition: Feature extraction, PCA, Wavelets
14. Pattern Recognition: Bayesian Inference, Linear classification

Biomedical Imaging

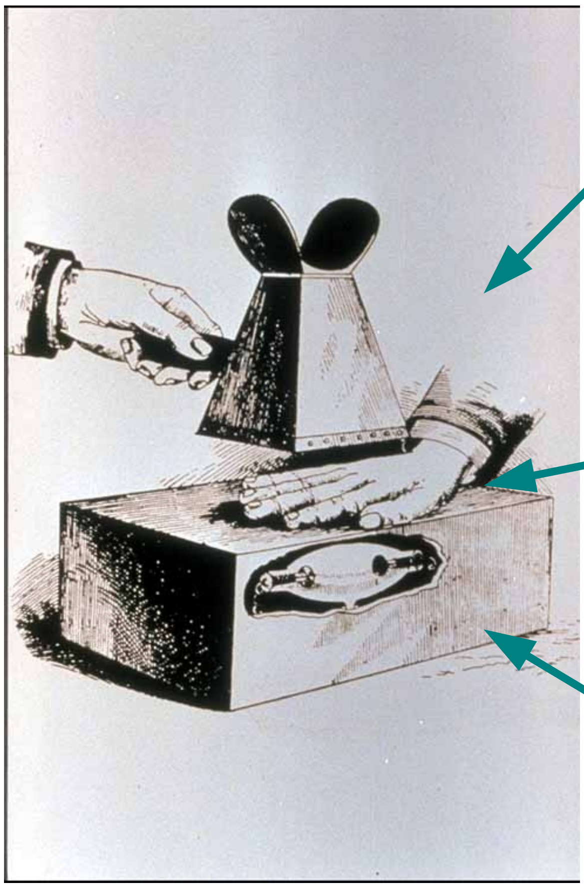
Imaging Modality	*Year*	*Inventor*	*Wavelength Energy*	*Physical principle*
X-Ray	**1895**	**Röntgen (Nobel 1901)**	**3-100 keV**	**Measures variable tissue absorption of X-Rays**
Single Photon Emission Comp. Tomography (SPECT)	1963	Kuhl, Edwards	150 keV	Radioactive decay. Measures variable concentration of radioactive agent.
Positron Emission Tomography (PET)	1953	Brownell, Sweet	150 keV	SPECT with improved SNR due to increased number of useful events.
Computed Axial Tomography (CAT)	1972	Hounsfield, Cormack (Nobel 1979)	keV	Multiple axial X-Ray views to obtain 3D volume of absorption.
Magnetic Resonance Imaging (MRI)	1973	Lauterbur, Mansfield (Nobel 2003)	GHz	Space and tissue dependent resonance frequency of kern spin in variable magnetic field.
Ultrasound	1940-1955	many	MHz	Measures echo of sound at tissue boundaries.

X-Ray Discovery

Wilhelm Conrad Roentgen (1845-1923) in 1896 and the first radiogram (of his hand) 1895:

Early X-Ray

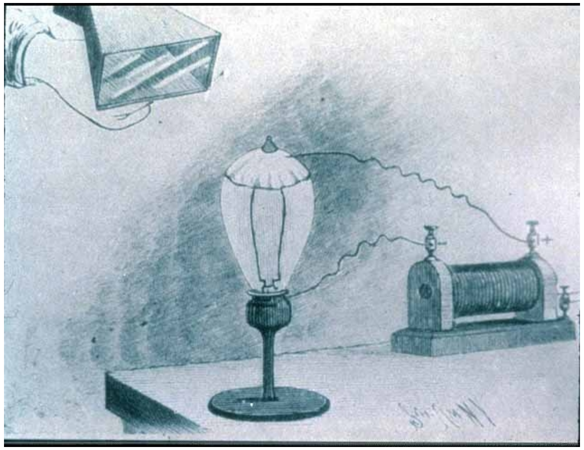
Schematic presentation of how it works:



Detection:
Fluorescent screen

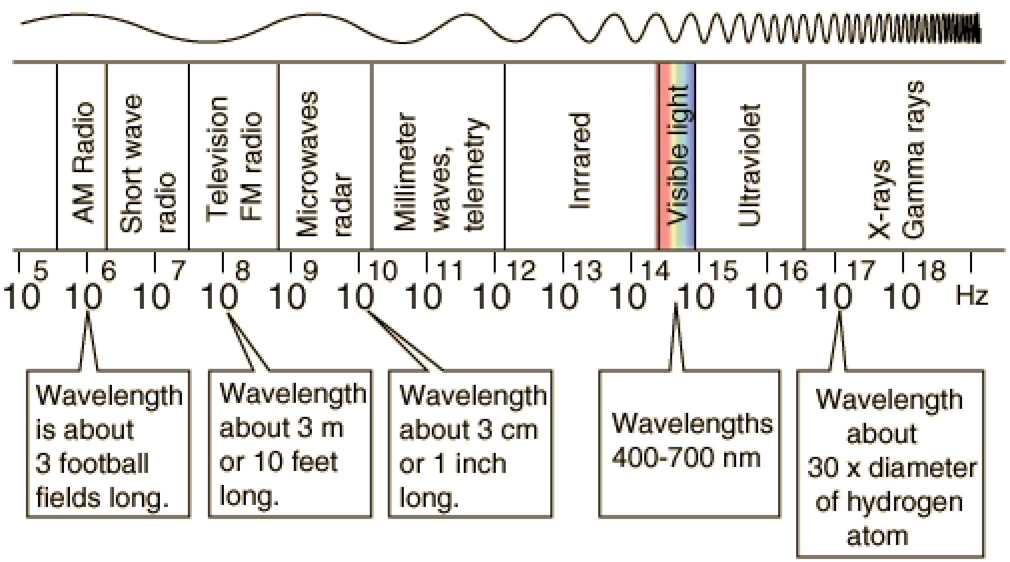
Interaction with tissue:
Absorption &
Scatter

Generation:
X-Ray tube



X-Ray Generation – Energy

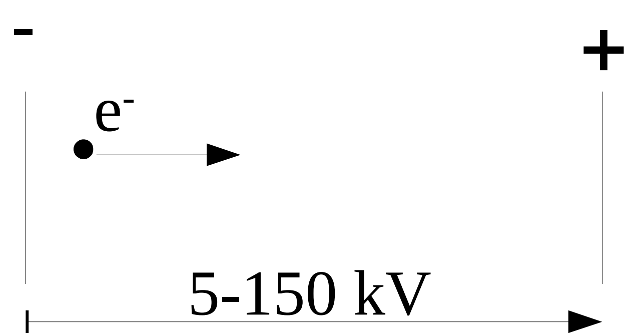
X-ray are high energy electromagnetic radiation above 3×10^{16} Hz and below 10 nm.



$$c = \lambda \nu$$

$$c = 3 \times 10^8 \, m/s$$

Energy in the keV range:

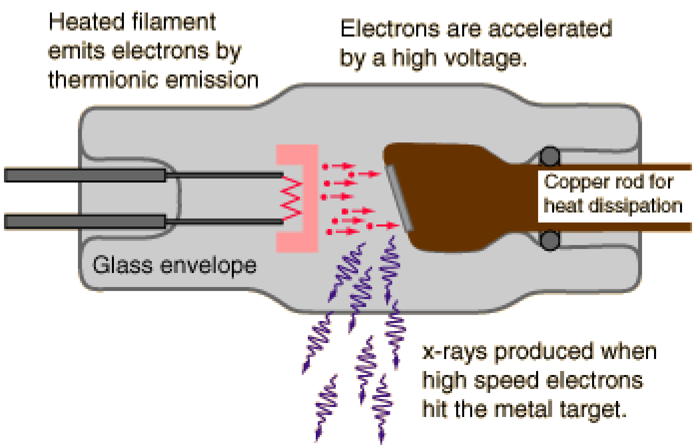


$$E = \nu h$$

$$h = 4.136 \times 10^{-15} \, eV \, s$$

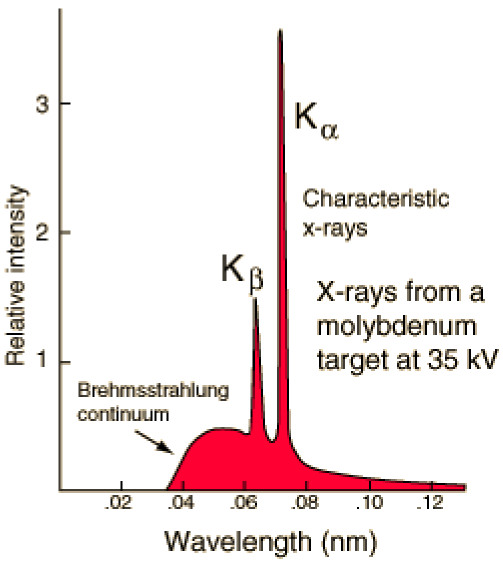
X-Ray Generation - Tube

X-ray vacuum tube accelerates electrons emitting form a heated cathode towards anode. When electrons impact on anode x-rays are emitted



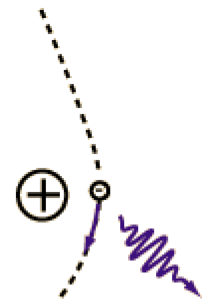
This leads to two forms of radiation:

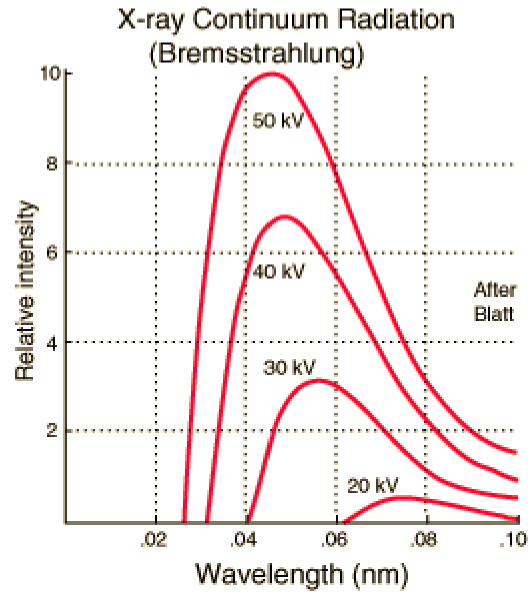
1. Bremsstrahlung or “breaking radiation”
2. Characteristic radiation



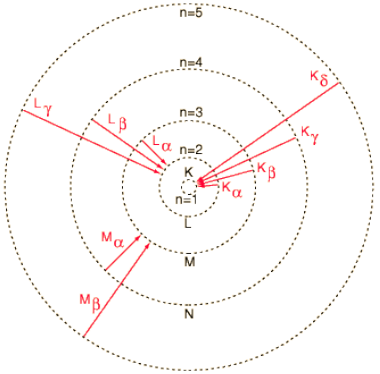
X-Ray Generation - Radiation

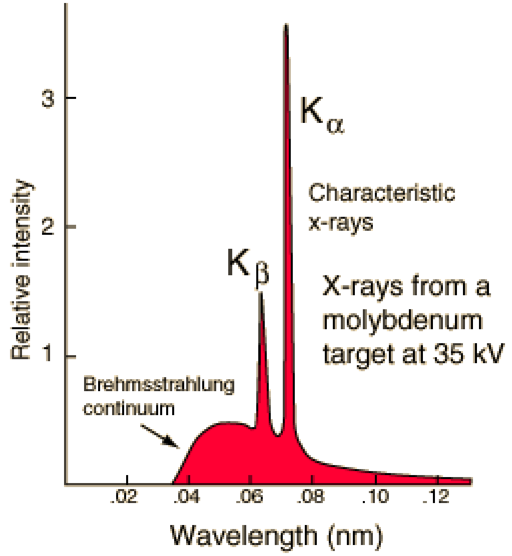
Bremsstrahlung:
Maxwell's equations imply that accelerated (or de-accelerated) charges emit radiation.





Characteristic radiation:
Electrons ionize atoms in anode. Radiation is emitted from heavy elements when their electrons subsequently make transitions between the lower atomic energy levels (K and L level) to fill that gap.





Figures from http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

X-Ray Generation – Tube Design

Rotating anode (typically Tungsten) is used to increase surface area and reduce heating.

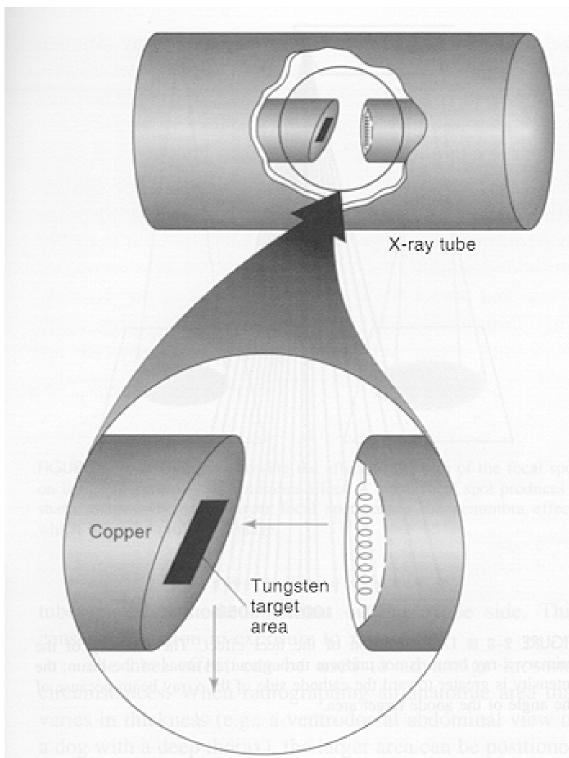


FIGURE 2–5 ■ Stationary anode construction.

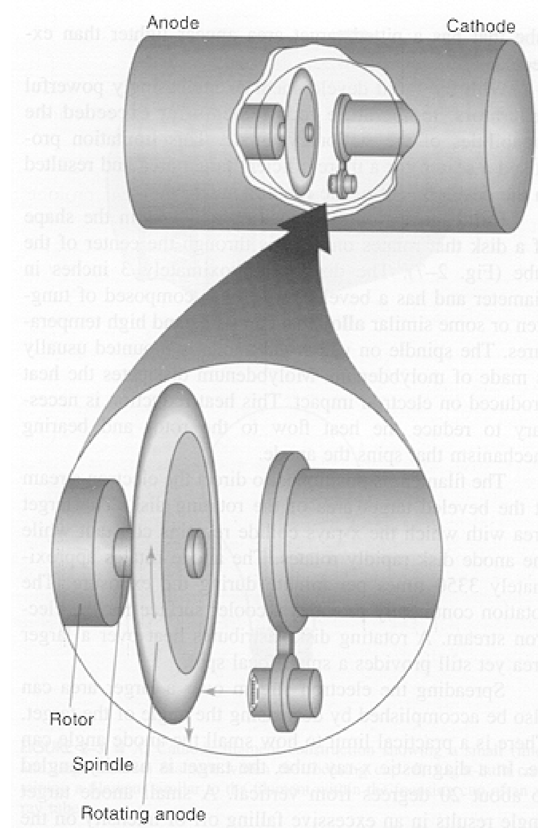


FIGURE 2–7 ■ Example of a rotating anode.

X-Ray Generation – Tube Design

Due to finite size of focal spot on the anode the image of a disk has a penumbra. This leads to blur in the final image, i.e. reduced spatial resolution. The goal is to reduce effective focal spot.

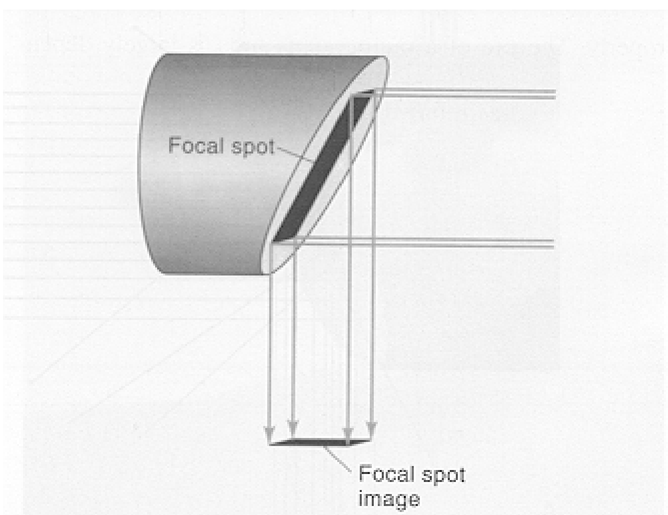


FIGURE 2–9 ■ The focal spot is the area in which the electrons collide with the target.

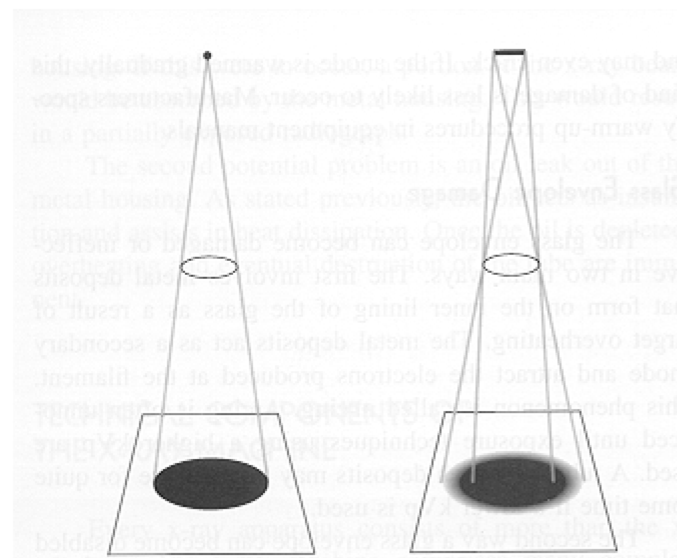
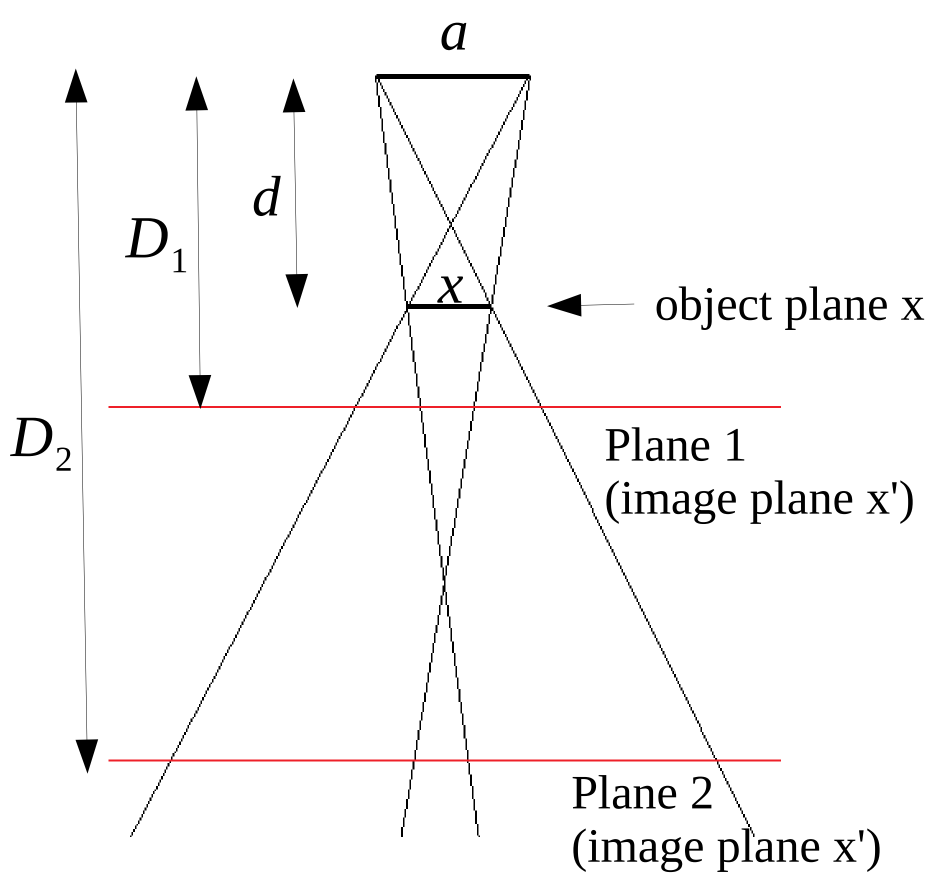


FIGURE 2–10 ■ Diagram showing the effect of the size of the focal spo on image sharpness—the penumbra effect. A small focal spot produces a sharp image, whereas a larger focal spot causes the penumbra effect which blurs the projected image.

Assignment: X-Ray Resolution



Q1: Given the geometry and variables, what is the FWHM of the image on the plane x'? (Hint: use similar triangles)

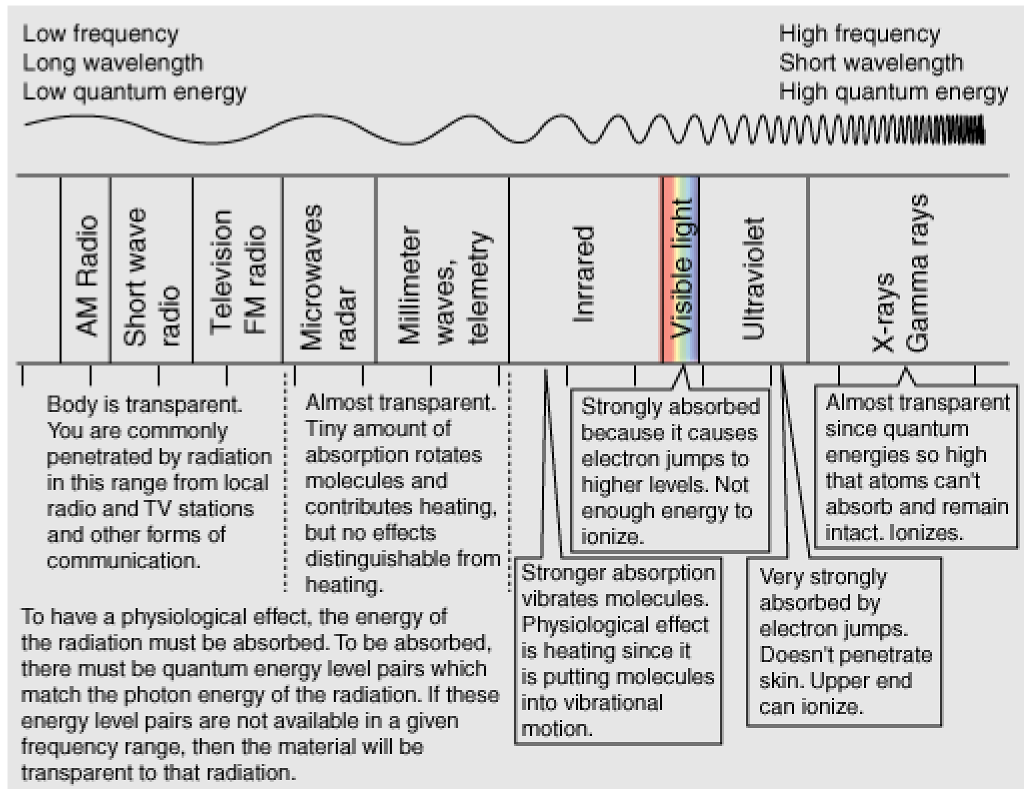
Q2: Add another object of length x on the object plane, and define Δx as the distance between the center of the two objects, what is the distance between the center of the two images on the plane x' (denoted as Δx')?

Q3: How small can Δx be such that the two images on plane x' are still resolvable? (Hint: the distance between two images Δx' cannot be smaller than the FWHM)

Q4: Will the answers to Q1~Q3 be different if we move the image plane x' from Plane 1 to Plane 2?

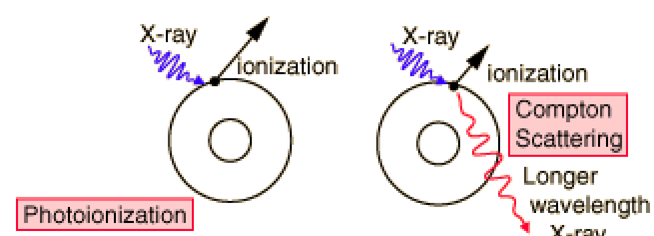
X-Ray Interaction with tissue – EM radiation

Interaction of EM waves with tissue:

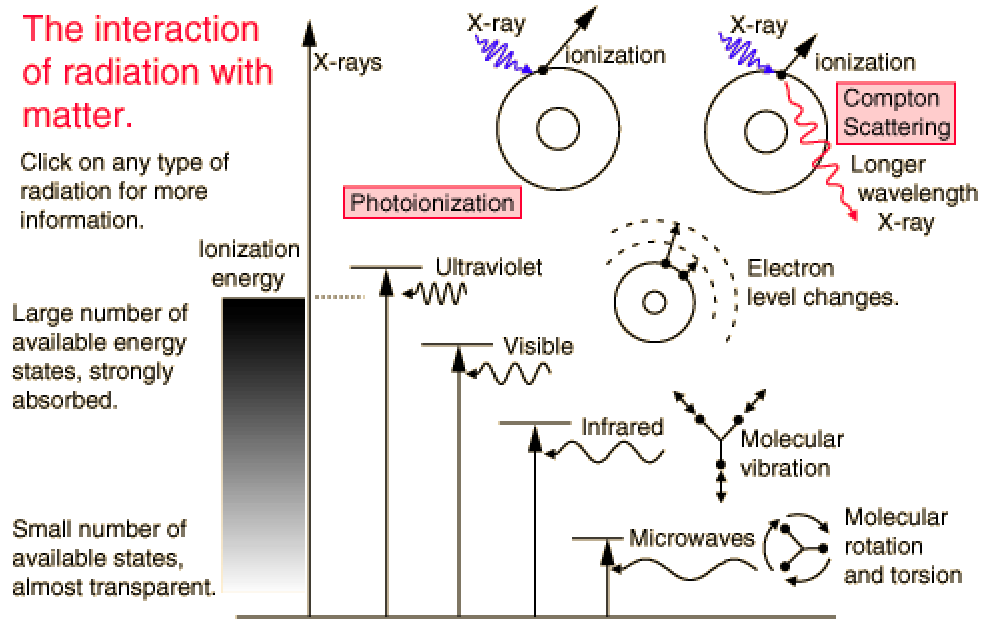


X-ray interact primarily through

1. Photoelectric absorption
2. Compton scattering



X-Ray Interaction – Scatter and Absorption



Interactions as photon progressively loose energy:

decreasing energy

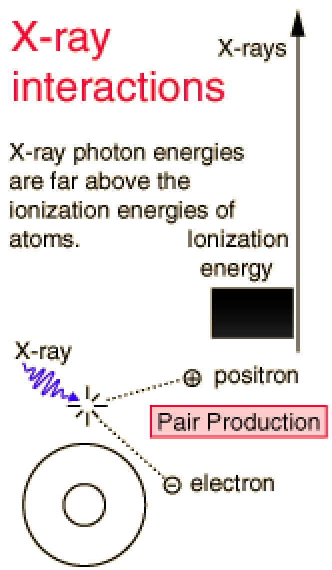
1. Pair production*
2. Compton scattering*
3. Photoelectric absorption*
4. Inelastic scattering
5. Non-ionizing absorption

* ionizing and therefore carcinogenic - bad!

X-Ray Interaction – Scatter and Absorption

Secondary photons after Compton scatter may scatter again and will eventually be absorbed.

1.



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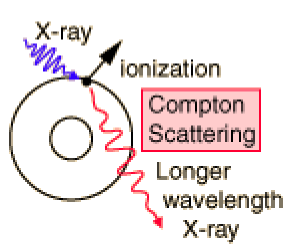

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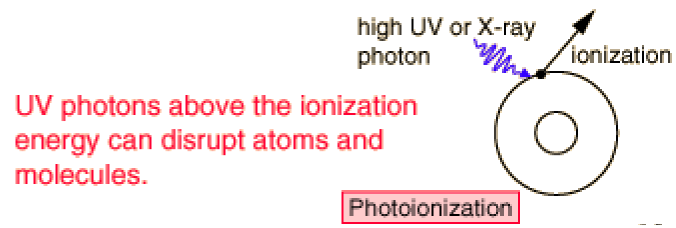

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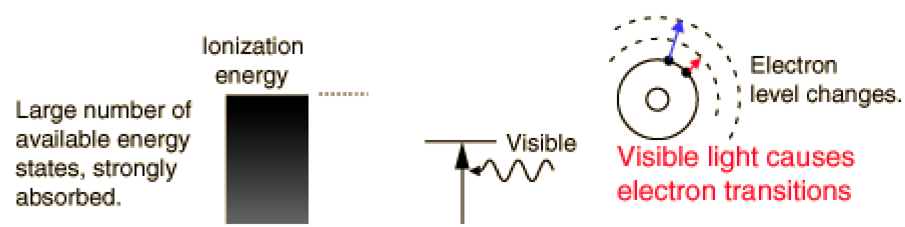

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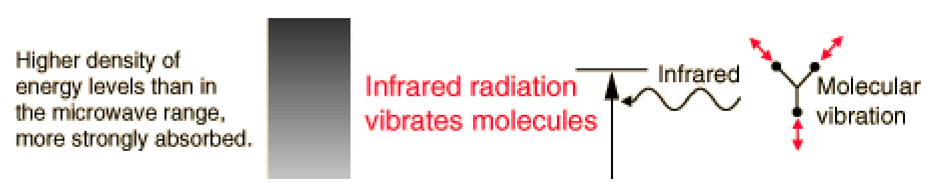
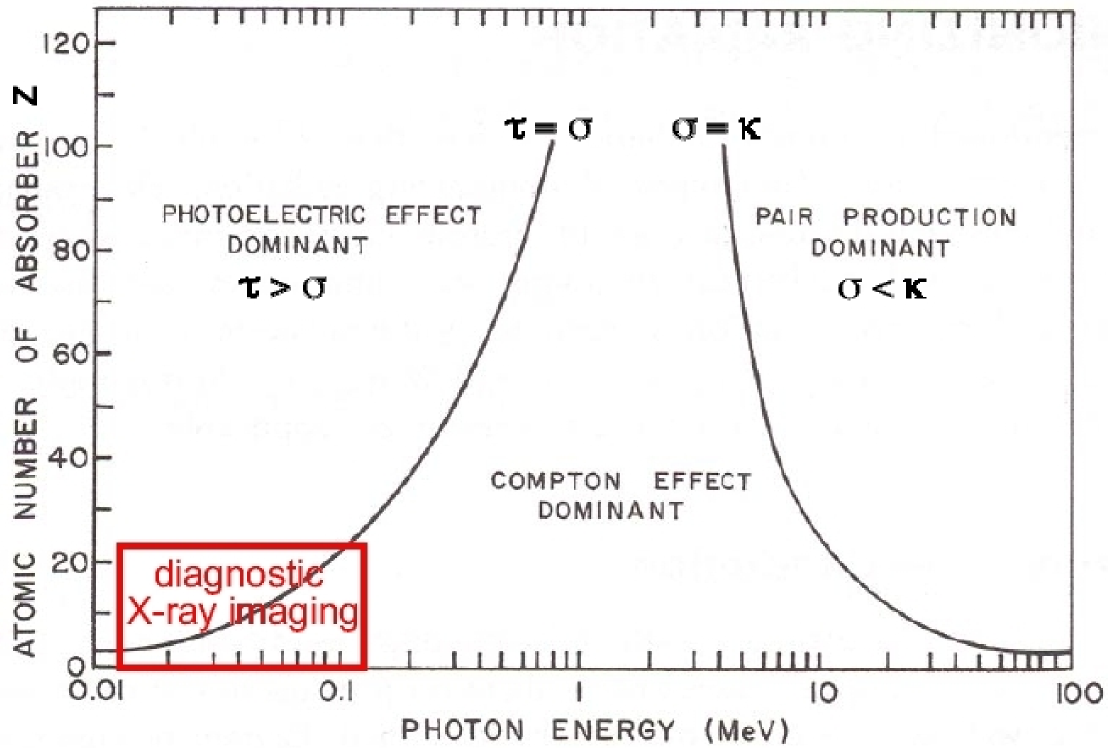


X-Ray Interaction – Scatter and Absorption

Attenuation coefficient is dominated by Compton scatter and photoelectric absorption. Likelihood of photo-electic effect τ, Compton effect σ, pair production κ.



Total likelihood of x-ray absorption:

$$\mu = \tau + \sigma + \kappa$$

X-Ray Interaction with tissue - Parameters

Likelihood of events are dominated by atomic number Z, photon energy E=hν, electron density ρ_e and mass density ρ:

Variables That Influence Principal Modes of Interaction of X and γ Rays

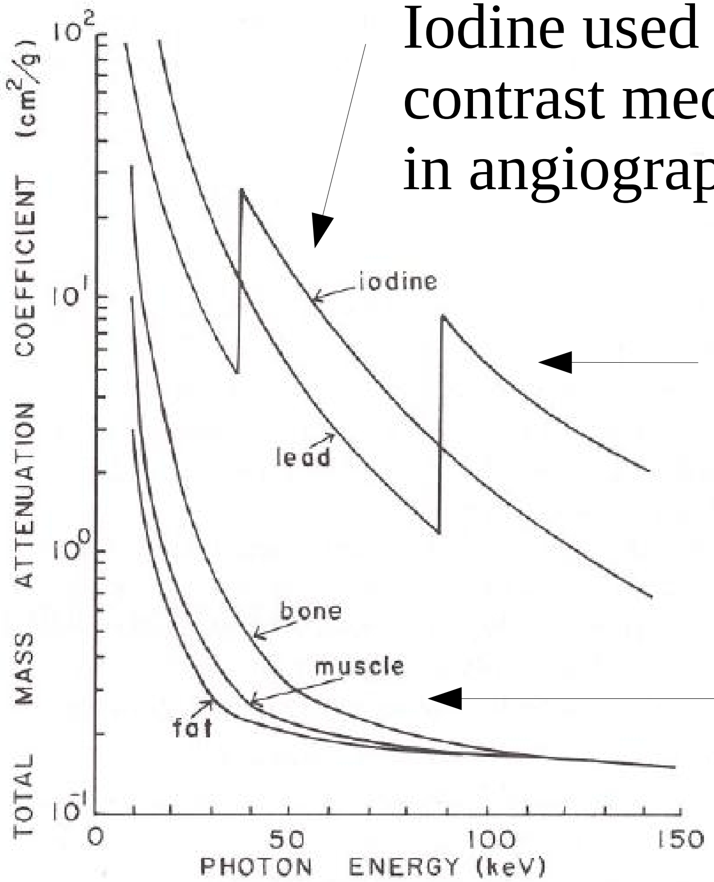
	Mode of Interaction	Dependence of Linear Attenuation Coefficient On: Photon Energy $h\nu$	Atomic Number Z	Electron Density ρ_e	Physical Density ρ
τ	Photoelectric	$\frac{1}{(h\nu)^3}$	Z^3	—	ρ
σ	Compton	$\frac{1}{h\nu}$	—	ρ_e	ρ
κ	Pair production	$h\nu$ (>1.02 MeV)	Z	—	ρ

Effective Atomic Number, Physical Density, and Electron Density for Air, Water, and Body Constituents

Material	Effective Atomic No.	Density (g/cm^2)	Electron Density (Electrons/kg)
Air	7.6		3.01×10^{26}
Water	7.4	1.00	3.34×10^{26}
Soft tissue	7.4	1.00	3.36×10^{26}
Fat	5.9–6.3	0.91	$3.34\text{–}3.48 \times 10^{26}$
Bone	11.6–13.8	1.65–1.85	$3.00\text{–}3.19 \times 10^{26}$

X-Ray Interaction – Attenuation Coefficient

Likelihood of scatter and absorption events depend on photon energy:



Iodine used as contrast medium in angiography

Material	μ(cm⁻¹) at 60keV
Air	0
Water	0.21
Muscle	0.2
Fat	0.18
Bone	**0.38**

Lead used for x-ray shielding

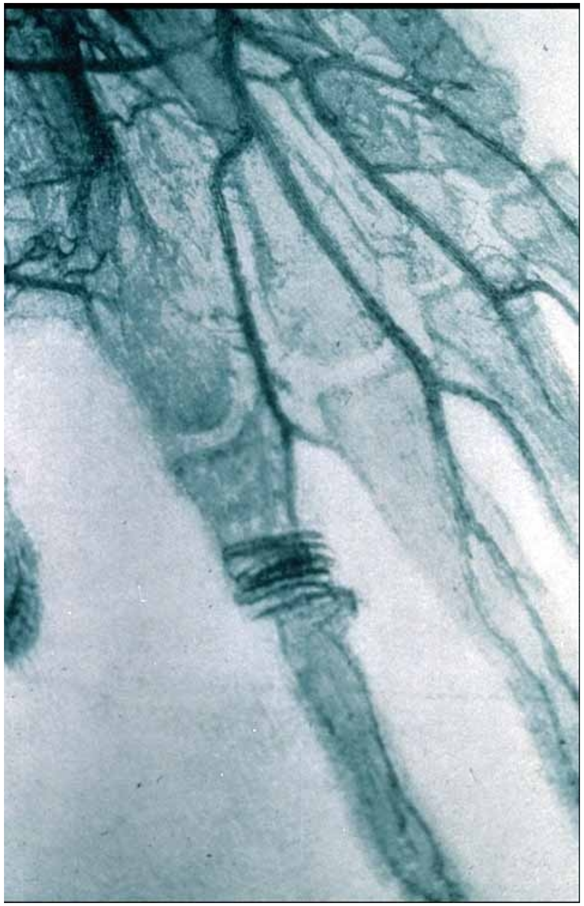
Notice contrast between bone and soft tissue.

Lower energies give better soft tissue contrast.

(Attenuation coefficient is sometimes given as a density to factor out the effect of mass density ρ)

X-Ray Interaction with tissue

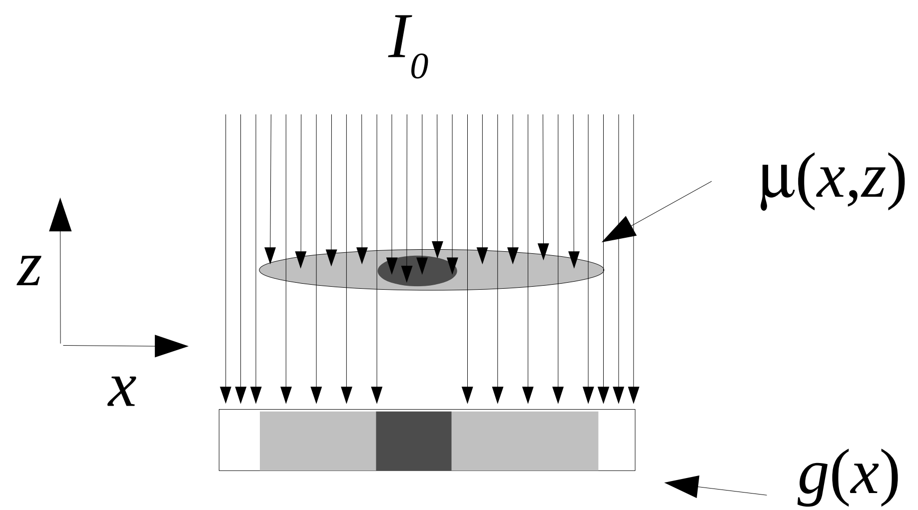
This first Angiography image of 1896 demonstrates well the contrast of due to high and low Z:



Post-mortem injection of mercury compounds (Haschek and Lindenthal of Vienna 1896).

X-Ray Interaction - Transmission Imaging

Spatially dependent attenuation coefficient and narrow parallel x-ray beams give 'negative' image of summed attenuation $\mu(x,z)$

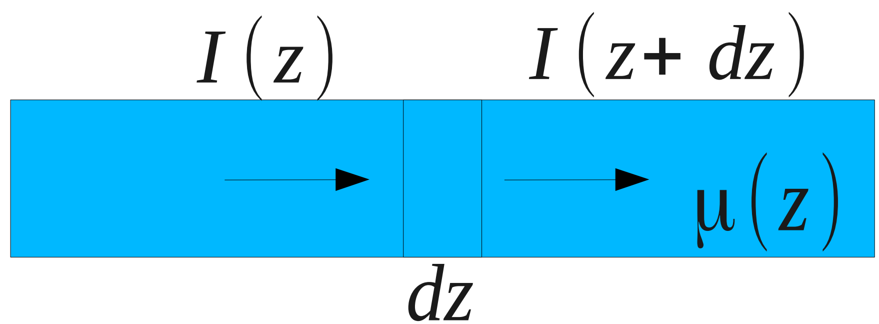


Ideal detector measuring log intensity combines attenuation linearly:

$$g(x) = -\log \frac{I(x)}{I_0} = \int dz\, \mu(x,z)$$

X-Ray Interaction - Transmission Imaging

X-ray imaging measures the intensity of light not absorbed or scattered by tissue, which is quantified by the cumulative **attenuation coefficient** μ:



μ = likelihood of absorption per unit length

$$I(z+dz) = I(z) - I(z)\mu(z)dz$$

$$\frac{dI(z)}{dz} = \lim_{dz \to 0} \frac{I(z+dz) - I(z)}{dz} = -I(z)\mu(z)$$

$$I(z) = I(0)\exp\left(-\int_0^z dz'\mu(z')\right)$$

X-Ray Detectors

- **Photographic film**: Converts x-ray into chemical process.
- **Phosphor screen**: Converts X-ray to visible light.
- **Image intensifier**: Increases light intensity to detect low dose in real time.
- **Digital detectors (CCD)**: Direct detection and digitization of x-ray to improves image quality.
- **Collimators**: Reduce scattering noise by filtering non parallel rays.

X-Ray Detectors – Photographic film

Converts x-ray into chemical process.

$$\mathrm{AgBr} + h\nu \rightarrow \quad \mathrm{Ag} + \mathrm{Br}$$

Development of film, e.g.

$$2\mathrm{Ag} + 2\mathrm{HCl} \rightarrow \quad \mathrm{H}_2 + 2\mathrm{AgCl}$$

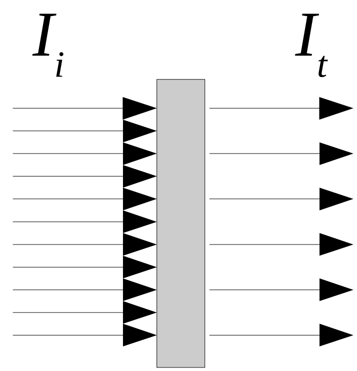
After exposing AgCl to metallic Ag dark spots appear (negative image).

Ideally the intensity of image should be logarithmic with incident x-ray intensity such that it measures the summed attenuation along a line.

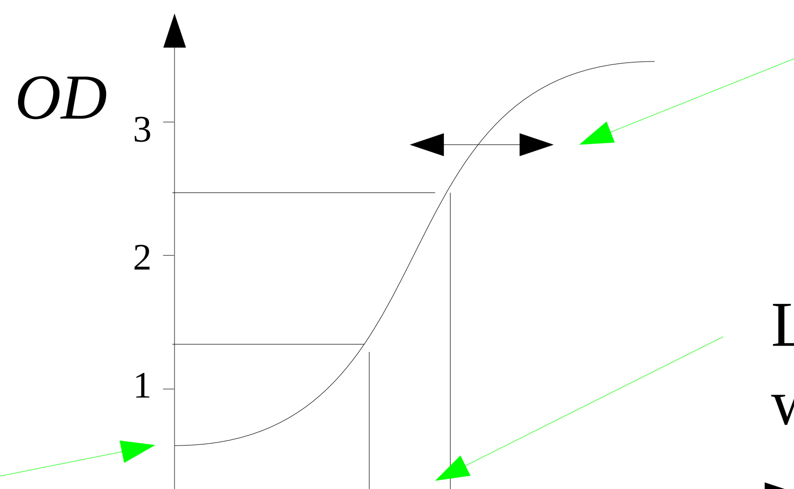
$$\ln \frac{I(x)}{I(0)} = -\int_0^x dx' \mu(x')$$

X-Ray Detectors – Photographic film

Optical density (OD) of film measures the amount of detected x-ray



$$OD = \ln \frac{I_i}{I_t}$$



Log exposure

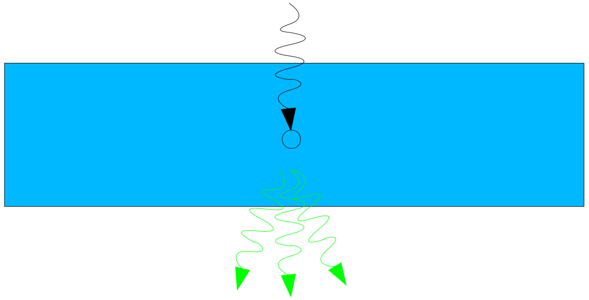
Speed: Affects required exposure times.

Latitude: linear range with 0.5 ≤ OD ≤ 2.5)

“fog”: background noise level

X-Ray Detection – Phosphor screen

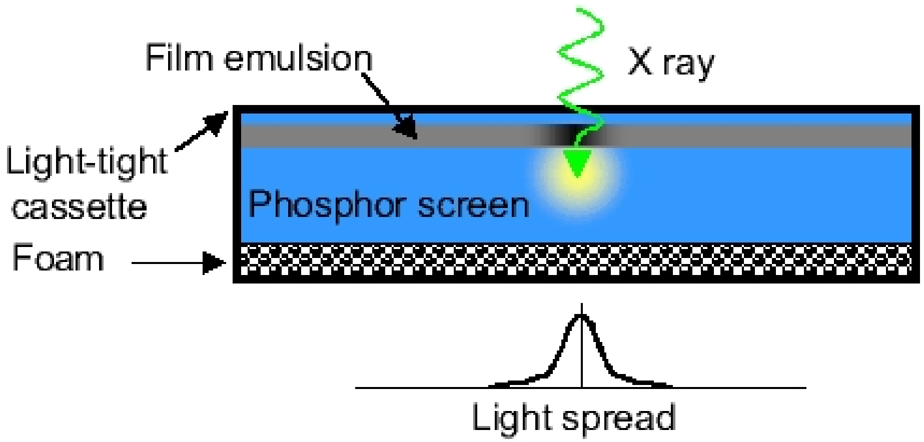
Converts X-ray to visible light.

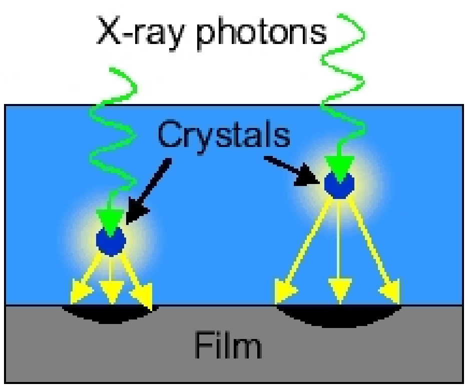


Rare earth elements (phosphors) absorb x-ray in photoelectric effect and emit energy as characteristic radiation in the visible range with some time delay ($>10^{-8}$ s). Single high energy x-ray photon is converted into many visible photons at lower energy.

X-Ray Detection – Phosphor screen

Phosphor screen increases the OD when combined with a photographic film

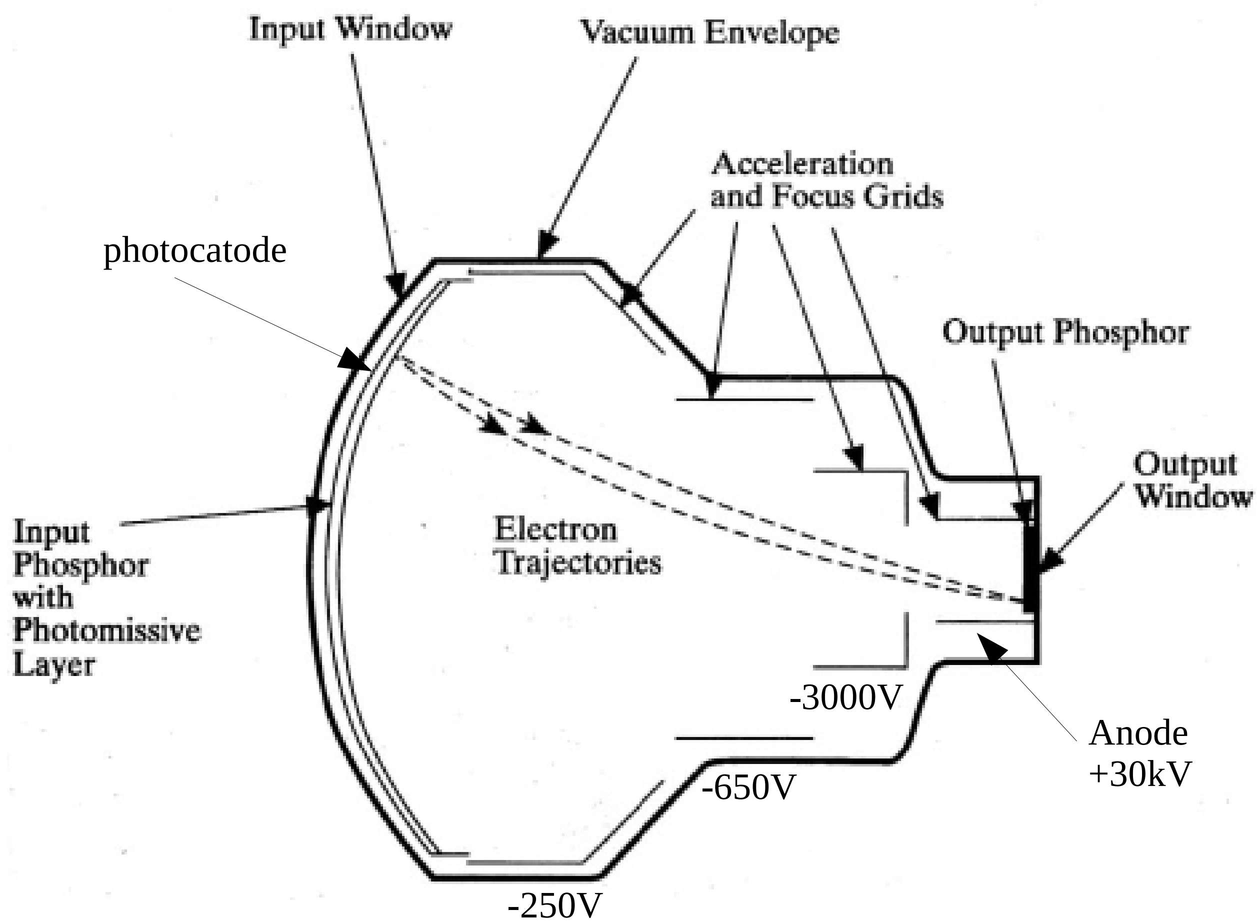




Notice tradeoff between sensitivity (thickness of phosphor screen) and resolution.

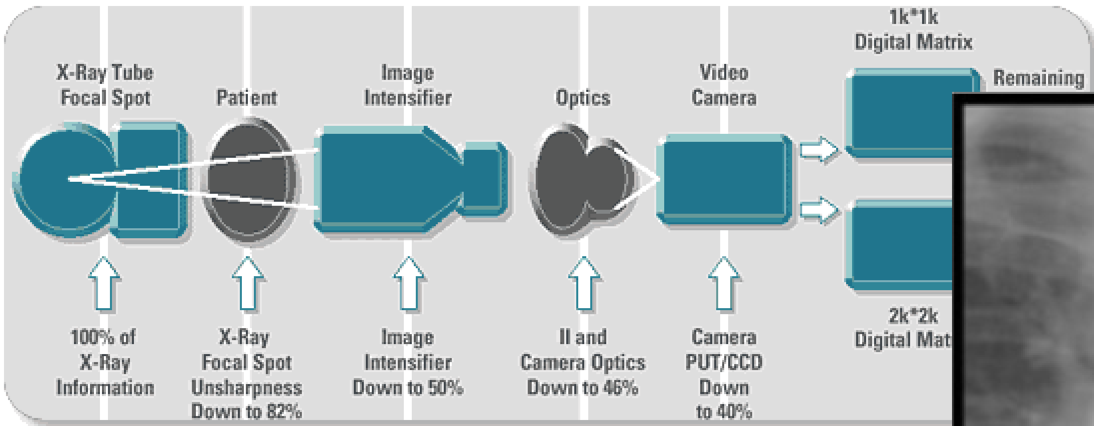
X-Ray Detection – Image Intensifier

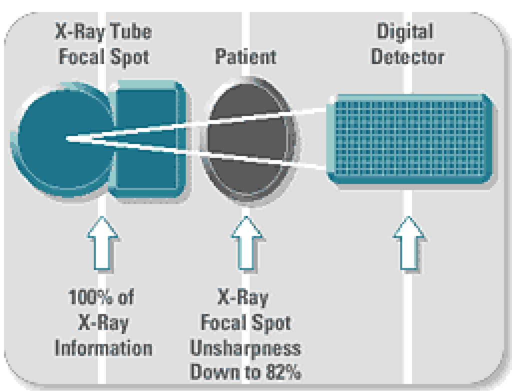
Increases light intensity to detect low dose in real time.
Signal is picked up by conventional digital camera.
Used for real time imaging (Fluoroscopy)

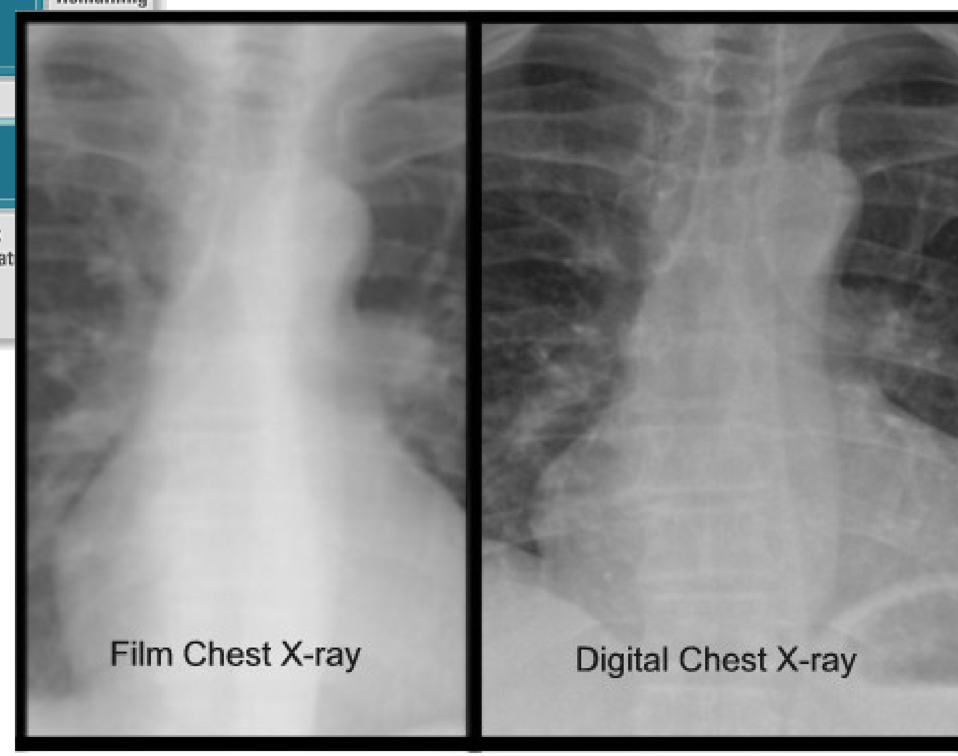


X-Ray Detection – Digital Detectors

Direct detection and digitization of x-ray to improves image quality by sidestepping sources of blur and noise.

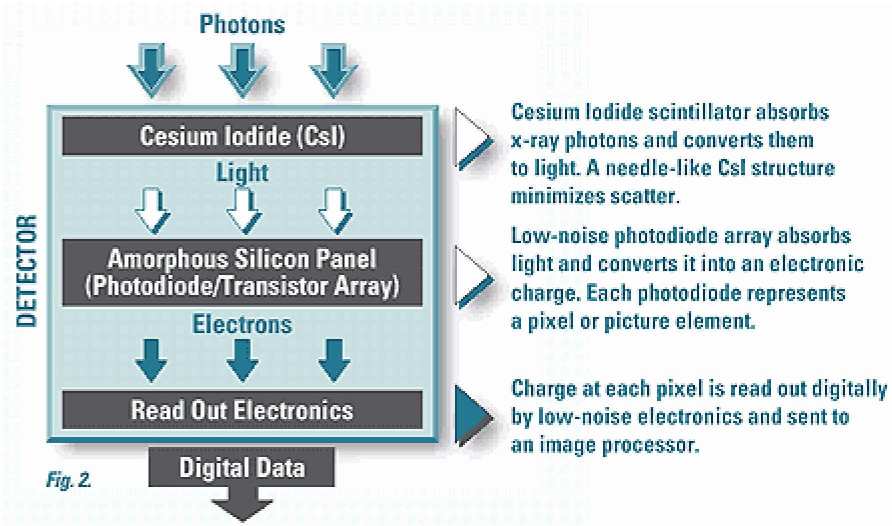


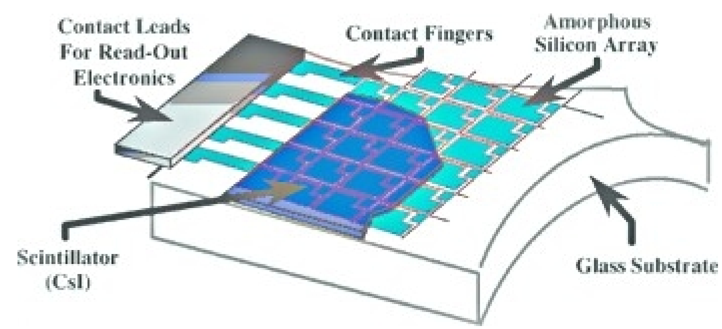


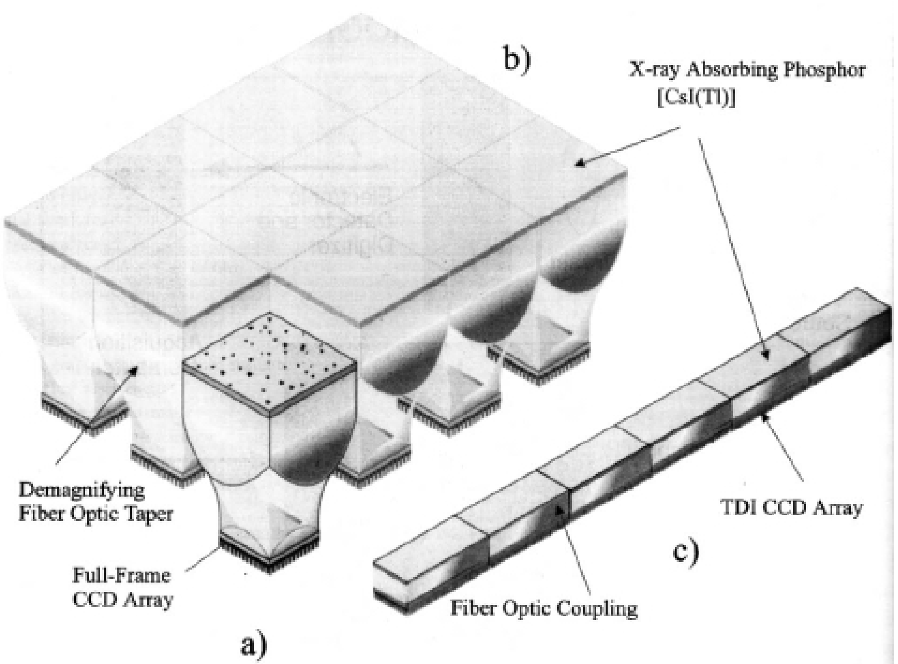


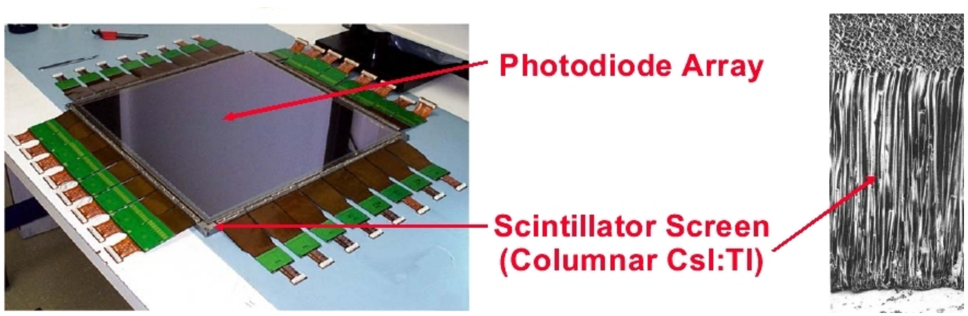
http://www.gemedicalsystems.com/rad/xr/

X-Ray Detection – Digital Detectors









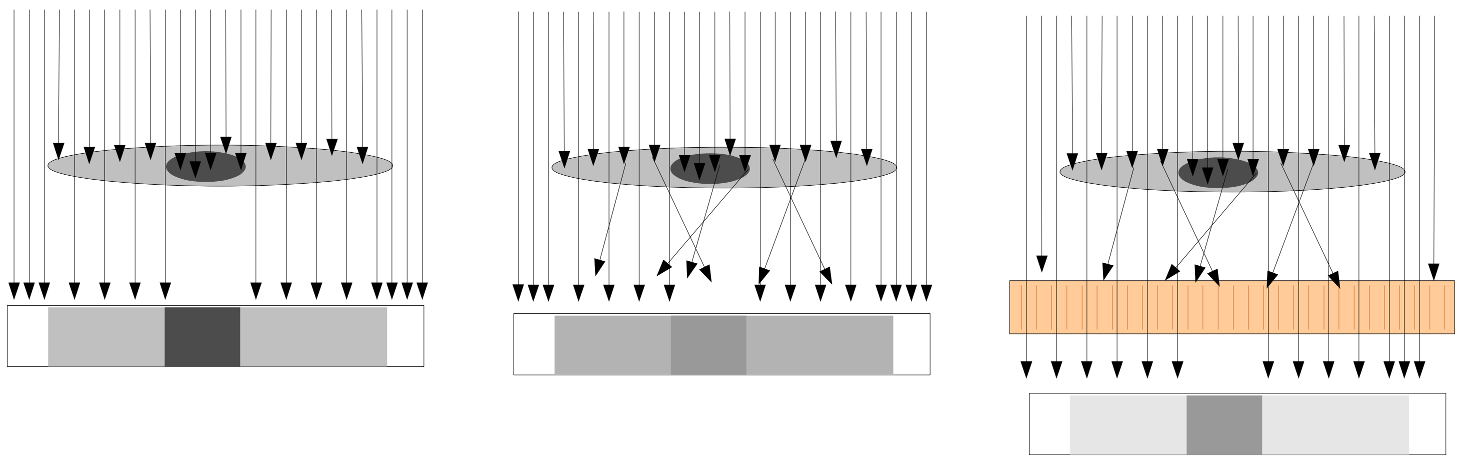
X-Ray Detection – Collimator

Reduce Compton scattering noise by filtering non-parallel rays with lead parallel holes collimator:

Ideally only absorption

Reduced contrast due to scatter

Scatter filtered by collimator



Disadvantage is reduced sensitivity.

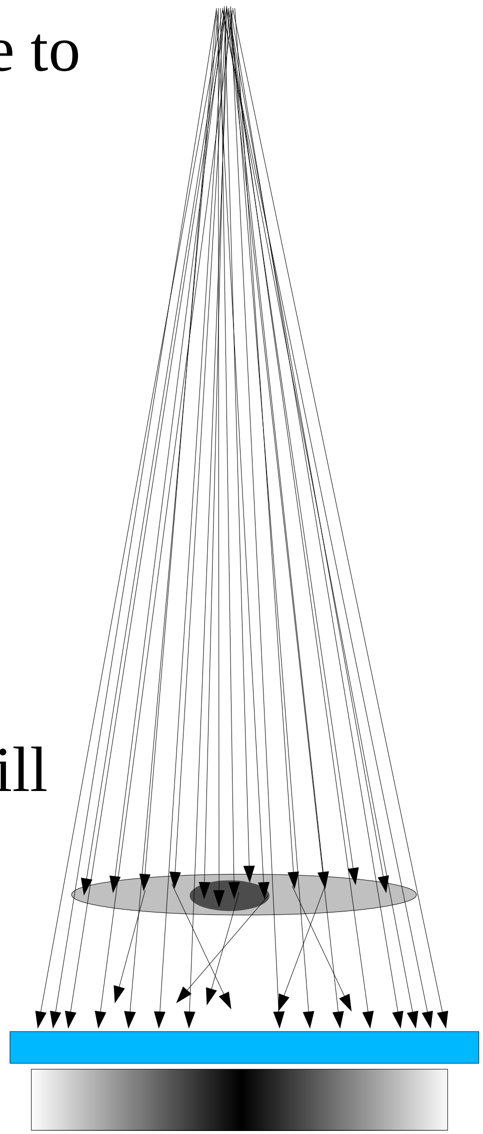
X-Ray Interaction - Transmission Imaging

In reality image blur has to be considered due to

- Finite focus of tube
- Compton Scatter
- Finite detector resolution (phosphor and film/CCD)

Consider a point like attenuation distribution $\mu(x)=\delta(x)$. The image of that point on the detector combines all blurring effects and is called the ***points spread function*** $h(x)$. We will assume that the system is linear and shift invariant (LSI).

$$h(x)=LSI[\delta(x)]$$



X-Ray Interaction - Transmission Imaging

LSI means that the PSF is **independent of the location** *x*

$$g(x) = LSI[\mu(x)] \quad \Rightarrow \quad g(x+x') = LSI[\mu(x+x')]$$

and the contributions from two points add **linearly:**

$$LSI[a\mu_1(x) + b\mu_2(x)] = a\,LSI[\mu_1(x)] + b\,LSI[\mu_2(x)]$$

The image of an arbitrary $\mu(x)$ is then given by a convolution

$$\begin{aligned} g(x) = LSI[\mu(x)] &= LSI\left[\int dx'\,\delta(x-x')\mu(x')\right] \\ &= \int dx'\,\mu(x')\,LSI[\delta(x-x')] \\ &= \int dx'\,h(x-x')\,\mu(x') \end{aligned}$$

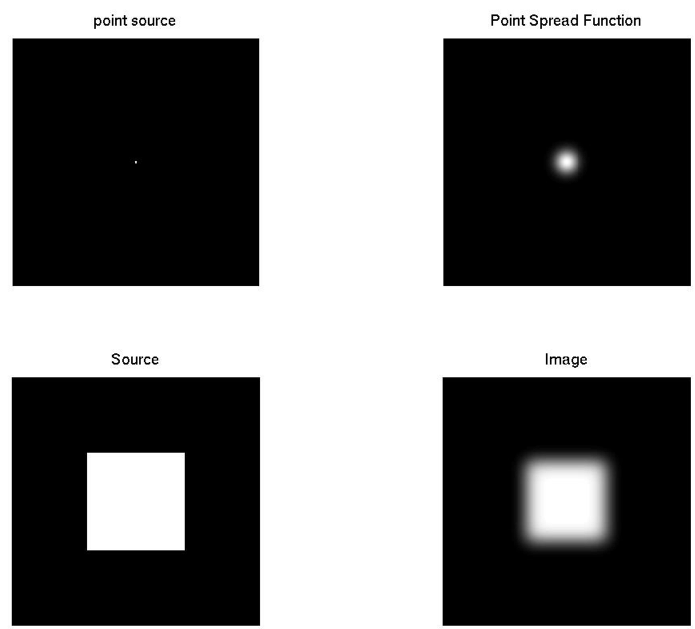
X-Ray Interaction - Transmission Imaging

For a 2D discrete array (image) we write the convolutions as

$$g(i,j) = \int_{x=1}^{N} \int_{y=1}^{M} h(i-x, j-y)\mu(x,y)$$

$$= h(i,j) * \mu(i,j)$$

```
>> g = conv2(mu,h);
```



X-Ray Interaction - Transmission Imaging

Anything that does not match these assumption can be lumped in to an additive contribution which we call noise $n(x)$

$$g(x) = n(x) + \int dx'\, h(x - x')\,\mu(x')$$

For a logarithmic detector the attenuation coefficients add up linearly in the image intensity.

Assignment 3:

a) Derive the image of a 1D occlusion. Include the effect of the penumbra due to the finite focus of the tube. Your result should be an analytic expression of the intensity distribution along the x axis.

b) Show whether this image is shift invariant or not.

c) compute the magnification factor. What does it depend on?

X-Ray Interaction – Beam Hardening

Attenuation is energy dependent, $\mu=\mu(\nu)$.

As lower energy x-rays are progressively absorbed due to larger μ the average energy or remaining x-rays increases. Effect is called "Beam Hardening" as higher energy rays have lower contrast on soft tissue and only show bone.

Detectors see the intensity summed over all frequencies:

$$I=\int d\nu\, I_i(\nu)\exp\left(-\int dx\,\mu(x,\nu)\right)$$

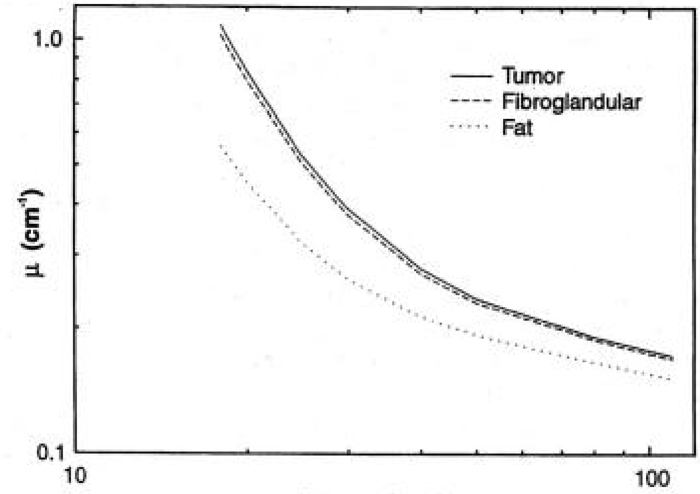
Energy dependent detector could correct beam hardening

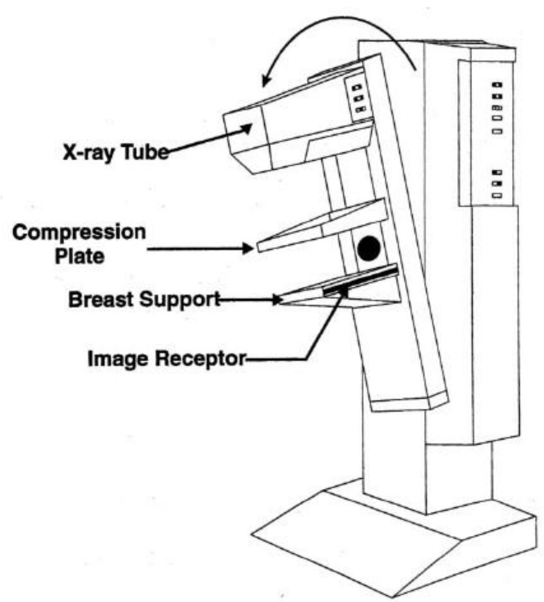
$$I(\nu)=I_i(\nu)\exp\left(-\int dx\,\mu(x,\nu)\right)$$

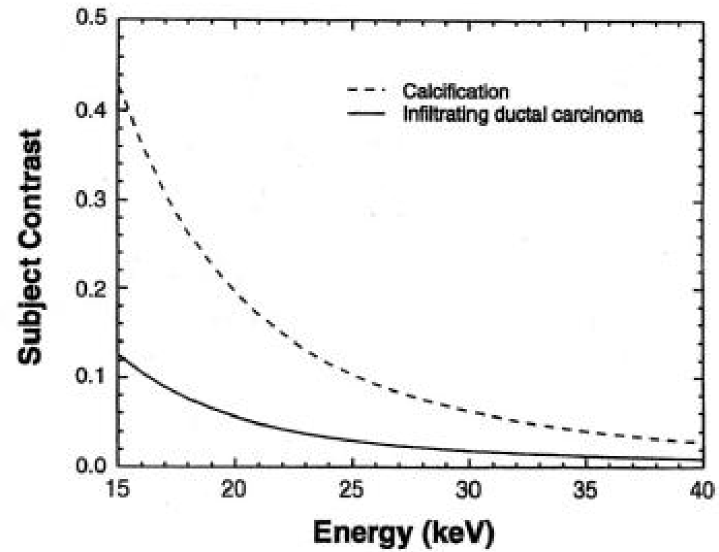
X-Ray Mammography

Low dose imaging at low energies to detect breast tumors at approx. 40 μm resolution.

- Soft tissue contrast best at low energies (18-23 keV)
- Collimator used to improve PSF and reduce background noise.
- Low dose to minimize seeding.





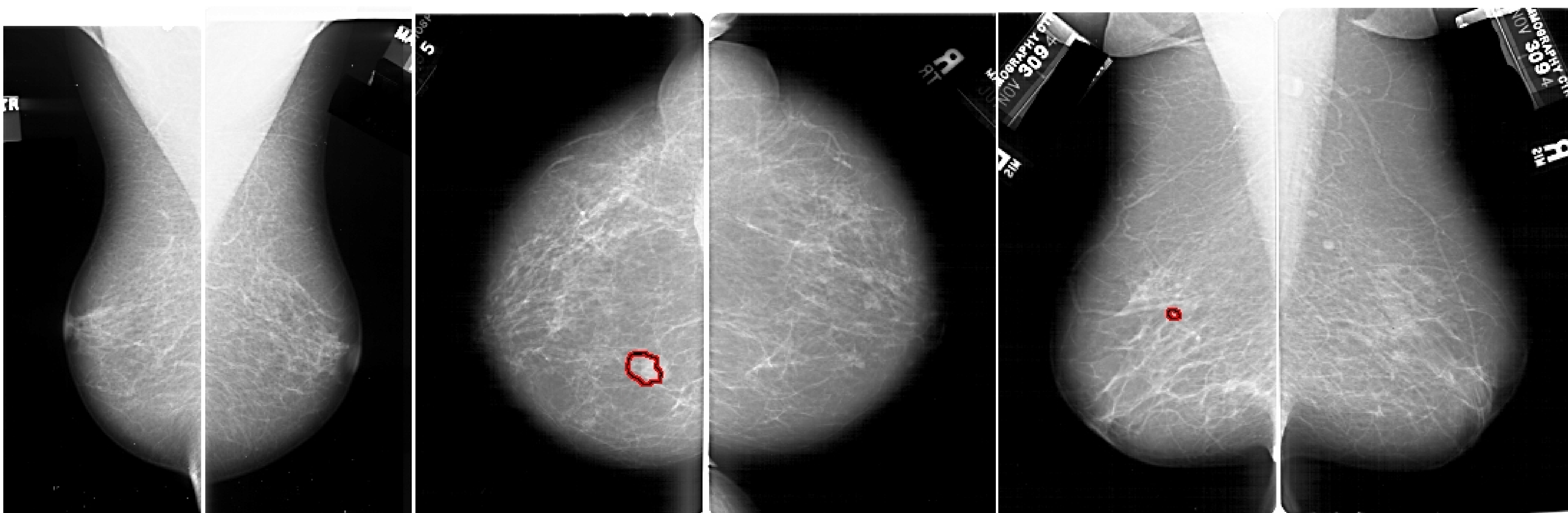


X-Ray Mammography

Tumor detection and diagnosis is difficult! It is based on:

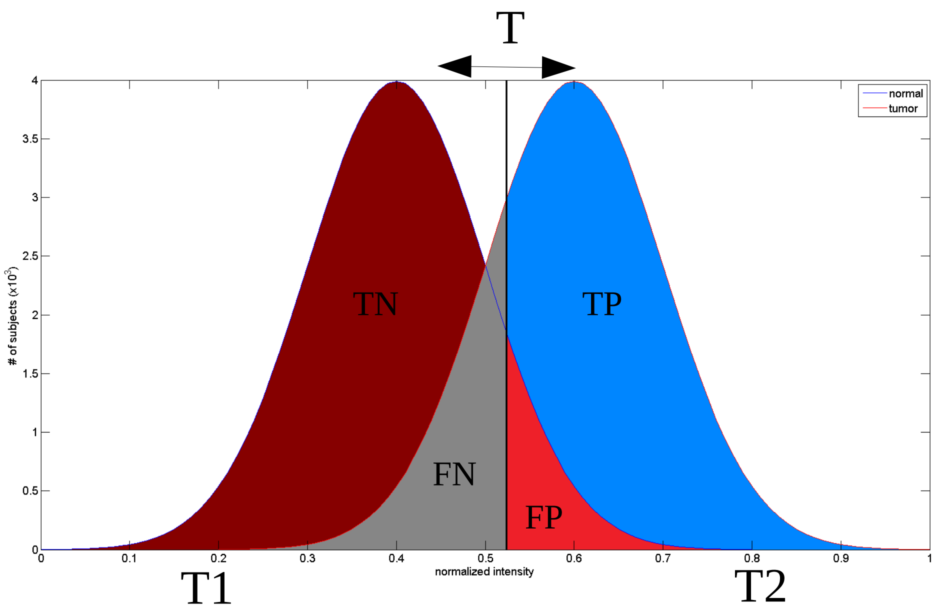
- characteristic morphology of normal tissue and tumor mass
- micro-calcifications
- asymmetry between left/right breast.

Normal Benign Tumor



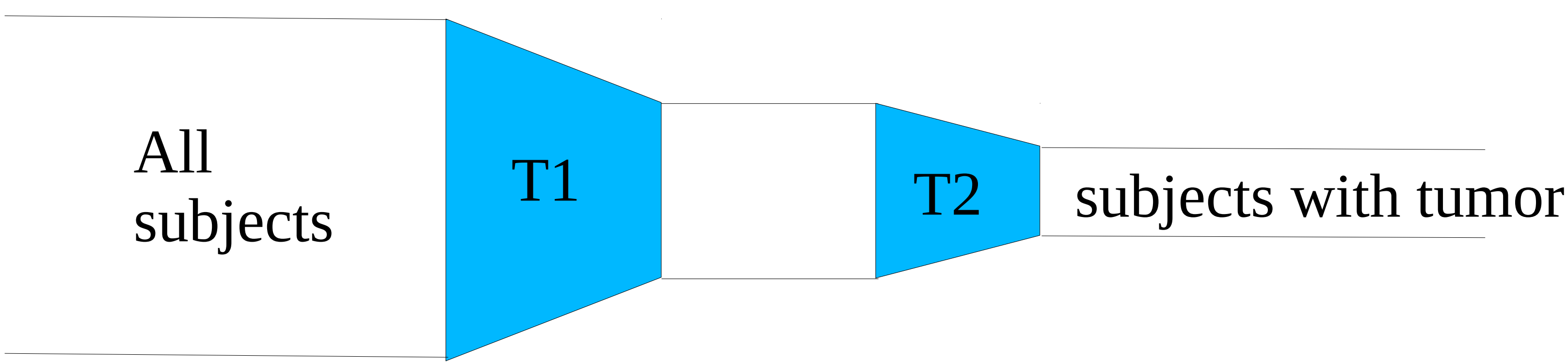
http://marathon.csee.usf.edu/Mammography/Database.html

Tumor Detection



TP + FN = 1
FP + TN = 1

T1: high sensitivity, low specificity
T2: low sensitivity, high specificity



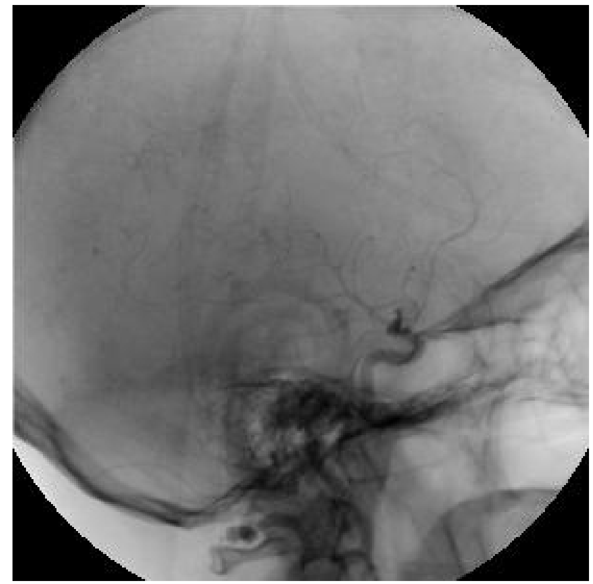
X-Ray Angiography

- Iodine compound injected as contrast agent to visualize blood vessels.
- Images at approx. 100 μm
- Short pulse to minimize motion blurring (10-100 ms depending on application)
- Most important application is the detection arterial obstructions.
- Also used in combination with fluoroscopy for real time monitoring of interventions such as angioplasty, catheter placement, etc.
- *Digital Subtraction Angiography* requires accurate (and flexible) registration of pre/post injection images.
- Composite images (on the left) also require accurate registration.

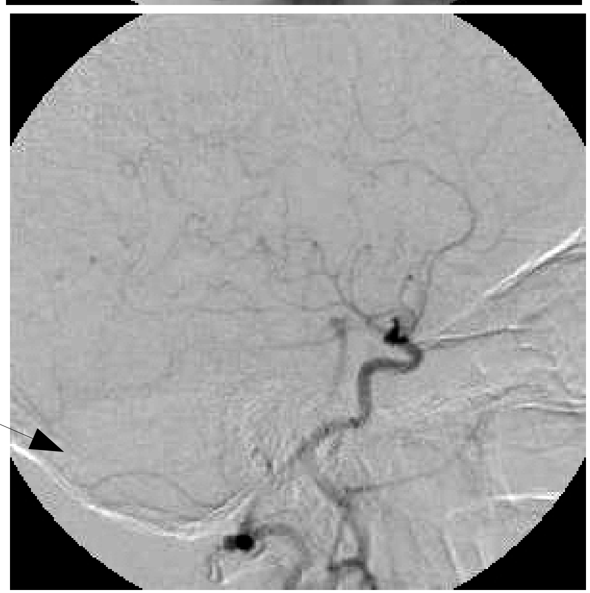
X-Ray Digital Subtraction Angiography

Pre-contrast

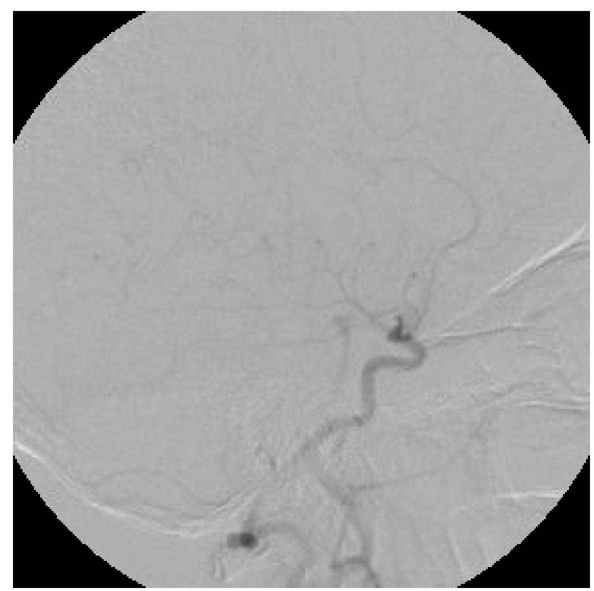
Post-contrast



Motion artifacts



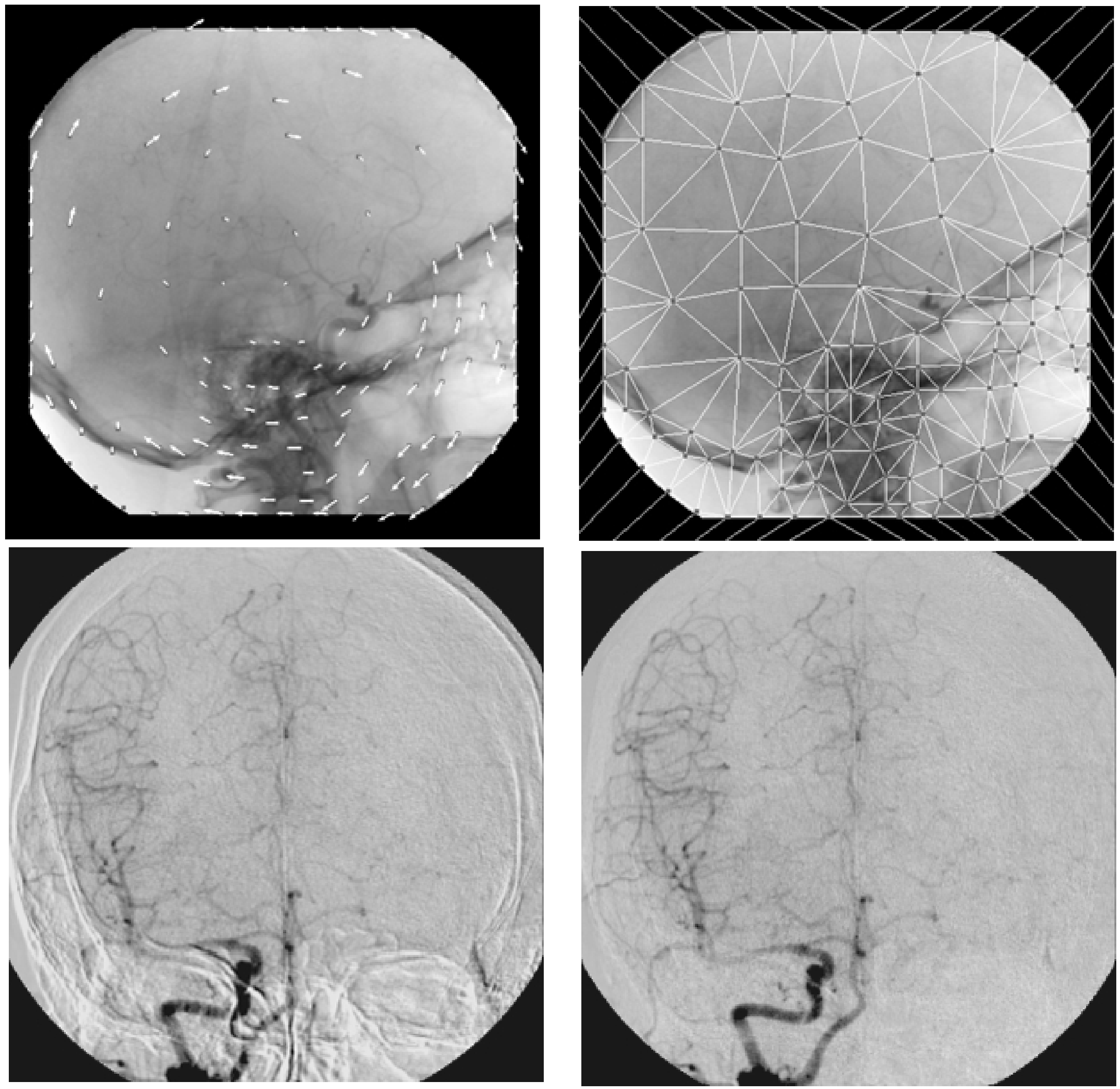
Contrast enhanced



Subtraction

X-Ray Digital Subtraction Angiography

Flow Motion | Warp patches



Before correction | After correction

http://imagescience.bigr.nl/meijering/research/registration/

Assignment: coregistration

This is a two part assignment (graded separately)

Part 1: Using interp2 to resample an image on a new rectangular grid with the same number of pixels (rows and columns) as the original image. Allow for the new grid to be scaled, shifted, and rotated (about the center of the image). This will implement zooming, panning, and rotating of an image. Summarize this code in a single function of the form

```
imgnew = mytransform(img,scale, xshift, yshift, alpha)
```

and demonstrate it on an image of your choosing. The goal of this assignment is to learn the use of 2D interpolation. Do not use matlab's image scale of rotate functions, use interp2 instead. Please submit a single function, along with a script that calls your function for some examples.

Part 2: Using the mytranform() developed in Part 1, implement coregistration of two images. To this end minimize the distance (mean square difference) between one image and the transformed version of the other. Use fminsearch() to find the optimal scale, shift, and rotation. Use multiple restart to find the best solution. Display the two images and their difference image after the optimal transform has been found. Use any two image you have taken of a same object (try to not vary illumination and view direction when taking the images.) Submit a single function with sub-functions (error function, transform function).

X-Ray Fluoroscopy

- Real-time x-ray imaging.
- Used in instruments during surgical interventions.
- Reduced x-ray intensity to minimize dose during continuous exposure.
- Therefore often contrast enhanced, e.g. blood vessels, and colon.

Example left: Air contrast Barium enema.