Syllabus

BME I5100: Biomedical Signal Processing and Signal Modeling
Prerequisites: complex variables, linear algebra, some programming, probabilities.
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Summary: This course introduces two fundamental concepts of signal processing: linear systems and stochastic processes. Various estimation, detection and filtering methods are developed and demonstrated on biomedical signals. The methods include harmonic analysis, auto-regressive model, Wiener and Matched filters, linear discriminants, and independent components. All methods will be developed to answer concrete question on specific data sets in modalities such as ECG, EEG, MEG, Ultrasound. The lectures will be accompanied by data analysis assignments using MATLAB.

Introduction
Linear, stationary, normal - the stuff biology is **not** made of.

Linear systems
Impulse response
Discrete Fourier transform and z-transform
Convolution
Sampling

Random variables and stochastic processes
Random variables
Moments and Cumulants
Multivariate distributions
Statistical independence and stochastic processes

Advanced topics
Introduction to causal inference
Introduction to deep learning

Examples of biomedical signal processing

Probabilistic estimation	
Linear discriminants	- **detection** of motor activity from MEG
Harmonic analysis	- **estimation** of hart rate in ECG
Auto-regressive model	- **estimation** of the spectrum of 'thoughts' in EEG
Matched and Wiener filter	- **filtering** in ultrasound
Independent components analysis	- **analysis** of MEG signals

Learning Outcomes:
- Practical programming skill in filtering and modeling biological signals.
- Practical programming experience in parameter estimation.
- Matlab programming skills
- Theoretical understanding of linear systems

Literature:

Eugene N. Bruce, Biomedical Signal Processing and Signal Modeling, John Wiley & Sons, 2000

Steven Kay, Fundamentals of Statistical Signal Processing, Prentice Hall, 1998

Monson H. Hayes, Statistical Digital Signal Processing and Modeling, John Wiley & Sons, 1996

Iranpour, R. and Chacon, P., Basic Stochastic Processes: The Mark Kac Lectures. MacMillan, 1988

Pearl, J. (2009). Causality (Cambridge: Cambridge University Press).

Francouis Chollet, Deep Learning with Python, 2nd Edition, Manning. 2021 (or latest version)

Week 1-4: Linear systems
The course will begin with and overview of properties of natural signals as a first approximation to the topics of linear systems, normal signals, and stationarity. Linear shift invariant systems will be introduced. This includes finite and infinite impulse response (FIR and IIR) and their representation in time, frequency and z-domain. Relevant concepts are magnitude and phase response, and poles and zeros. The convolution theorem for the Fourier and z-transform will be presented. The practical importance of the discrete Fourier transform for the implementation of linear and circular convolution will be explained. Issues related to discrete versus continuous time and finite versus infinite time will be discussed and in particular the practical importance of the sampling theorem.

Week 1:
Properties of biological signals: non-stationary, non-linear, non-Gaussian
Linear shift invariant system
Finite and infinite impulse response
Auto-regressive and moving average filters

Week 2:
Discrete Fourier transform and z-transform*
Magnitude and phase response
Poles and zeros
Stability

Week 3:
Convolution theorem
Linear versus circular convolution
Overlap-save implementation of linear convolution
Windowing

Week 4:
Discrete versus continuous time signals
Sampling theorem
Pre-filtering
Down-sampling

Week 5-8: Random variables and stochastic processes
Probability distribution and density function (pdf) for discrete and continuous variables will be introduced starting with one-dimensional random variables. Conditional distribution and additive random variables will be discussed. The importance of the normal distribution and the central limit theorem will be highlighted. Moments and Cumulants, and the characteristic function will be presented as ways of specifying a pdf. Distributions frequently discovered in biomedical signal processing such as Gaussian, Poison, and Lapacian will be presented. Multivariate distributions in particular Gaussian will be discussed, and the important concept of statistical independence, probabilistic inference, and Bayes rule. Finally the relationship of the material discussed to stochastic processes, i.e. Markov and Wiener process will be presented.

Week 5
Probability distribution and density function of 1D random.
Conditional distribution and additive random variables
Normal distribution and the central limit theorem.

Week 6
Moments and Cumulants
Characteristic function
Gaussian, Poison, and Lapacian

Week 7
Multivariate distributions
Covariance
Multivariate Gaussian
Product and convolutions of Gaussians
Conditional Gaussian (Shurr complement)

Week 8
Statistical independence, factorization
Bayes rule, prior, posterior
Probabilistic inference
Markov and Wiener process
Correlation, drift and variance

Week 9-14: Examples of biomedical signal processing
Maximum likelihood (ML) and maximum *a-posteriori* (MAP) estimation will be presented as it represents the basis for most of the methods to be developed. Binary linear classification will be explained on the task of predicting motor action from MEG signals. Issues concerning statistical performance and over-training will be highlighted. Harmonic analysis will be derived from ML and demonstrated on ECG hart rate estimation. Estimation of the auto-regressive parameters will be derived from ML. The relationship to linear prediction will be highlighted. The relevance for spectral estimation and an application to spectrum identification in EEG will be presented. Matched filter for detection and Wiener filtering for noise reduction will be presented and demonstrated on Ultrasound data. Finally an overview of linear decomposition methods such as Wavelets, principal components and independent components will be given. The focus will be placed on independent component analysis (ICA) and its application to the analysis of MEG signals.

Week 9:
Probabilistic estimation
Maximum Likelihood
Maximum a-posteriori estimation

Week 10
Linear discriminants - **detection** of motor activity from MEG

Logistic regression
ROC curve
Test versus training set performance

Week 11
Harmonic analysis - **estimation** of hart rate in ECG
Hart rate monitoring
Pitch detection

Week 12
Auto-regressive model - **estimation** of the spectrum of 'thoughts' in EEG
Linear prediction
Spectral estimation

Week 13
Matched and Wiener filter - **filtering** in ultrasound

Week 14
Independent components analysis - **analysis** of MEG signals
Wavelets, PCA, ICA

Assignments

Lectures will be accompanied by a data analysis assignment. The results will be discussed at the beginning of the following lecture. Example programs will be given in order to gradually introduce new concepts of the MATLAB programing language. The intention of the assignments is to give hands-on experience with data analysis and allow to explore the effects of different parameters in the analysis. This is not a programming course and therefore there will be no particular emphasis on efficient or elegant programming.

Note

This syllabus covers probably too much material. However, no material will be carried over from one week to the next. Instead, the material will be cut short dynamically depending on the amount the students can assimilate in each class.