BME I5100: Biomedical Signal Processing
(Was Non-linear signal processing in biomedicine)

Introduction

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Biomedical Signal Processing - Content

We will cover basic principles of signals processing. We will emphasize examples and focus on electrical signals generated by the biological systems (biopotentials).

We will introduce concepts from:

- filter theory
- statistical processes
- pattern recognition
- information theory
- probabilistic modeling
- Neurophysiology
- **MATLAB**

Prerequisite:

linear algebra, some programing language, complex variables.

Literature

Eugene N. Bruce, Biomedical Signal Processing and Signal Modeling, John Wiley & Sons, 2000

Schedule

Week 1: Introduction
Linear, stationary, normal - the stuff biology is **not** made of.

Week 1-4: Linear systems (mostly discrete time)
Impulse response
Moving Average and Auto Regressive filters
Convolution
Discrete Fourier transform and z-transform

Week 5-7: Random variables and stochastic processes
Random variables
Multivariate distributions
Statistical independence

Week 9-14: Examples of biomedical signal processing
Probabilistic estimation
Linear discriminants - **detection** of motor activity from MEG
Harmonic analysis - **estimation** of hart rate in Speech
Auto-regressive model - **estimation** of the spectrum of 'thoughts' in EEG
Independent components analysis - **analysis** of MEG signals
Klaman Filtering – motion estimation

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Week 8: Electrophysiology
Origin and interpretation of Biopotentials

Week 9-14: Examples of biomedical signal processing
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Biomedical Signal Processing and Signal Modeling

Biomedical Signal Processing -Signal processing and statistical modeling methods useful when analysing biomedical signals, e.g.

- Electro and Magneto Encephalography
- Electro Myograms and Cardiograms
- Circadian rhythm in body temperature
- Spike trains
- Speech
- ...

Property of BioMed signals: **non linear**, **non stationary**, **non Gaussian**

Linear, Stationary, Normal - *The stuff biology is not.*

Linear transformation $\boldsymbol{y} = L[\boldsymbol{a}]$:

$$y(t) = L[a\,x_1(t) + b\,x_2(t)] = a\,L[x_1(t)] + b\,L[x_2(t)]$$

Physics often calls for linear combination of signals:

- Mass, force, energy
- Concentrations in solutions.
- Electrical and magnetic fields.
- Intensity of incoherent electromagnetic radiation (X-ray, visible light, radio-waves)
- Amplitude of acoustic signal.
- ....

Lets look for example at EEG

```
>> load eeg.mat
```

Linear, Stationary, Normal *- The stuff biology is not.*

Example: We record frontal EEG electrode $y(t)$. It will be contaminated with eye muscle activity. Assume eye muscle activity generates electrical source signal, x_1(t), and some other frontal brain activity gives source, x_2(t). Physics tells us that electrical potentials add up linearly:

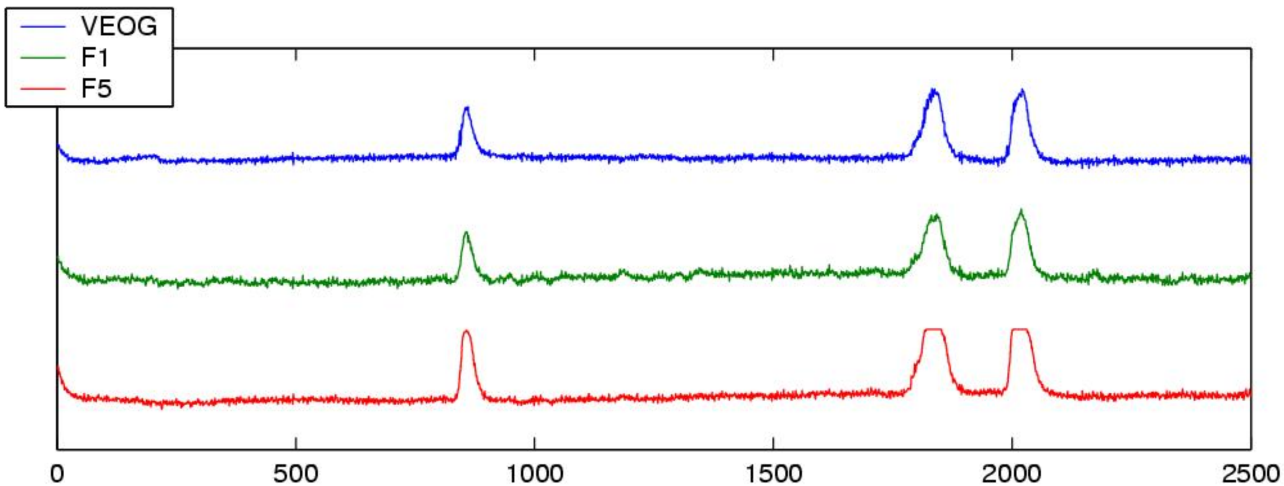
$$y(t) = [a\, b] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where [a b] represent the coupling coefficients for eye muscle and frontal activity respectively.

$$y(t) - a\, x_1(t) = b\, x_2(t)$$

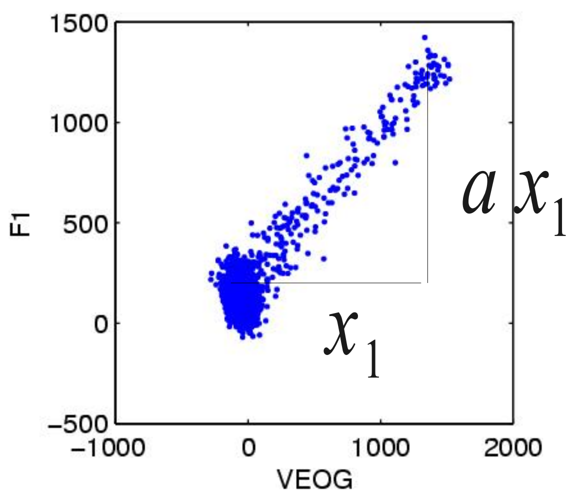
Linearity is crucial because given an estimate of $x_1(t)$ and a for example from an electro-oculogram (EOG) we can subtract its influence on $y(t)$:

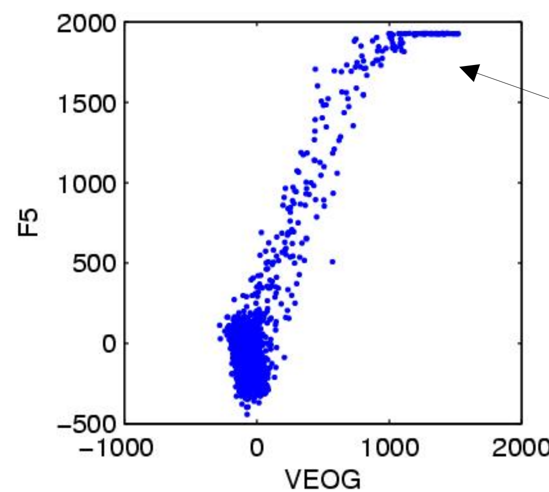
Linear, Stationary, Normal *- The stuff biology is not.*




```
>> plot([-x(1,:); x(8,:); x(2,:)]')
```

Unfortunately signals are often **distorted by non-linearity.**





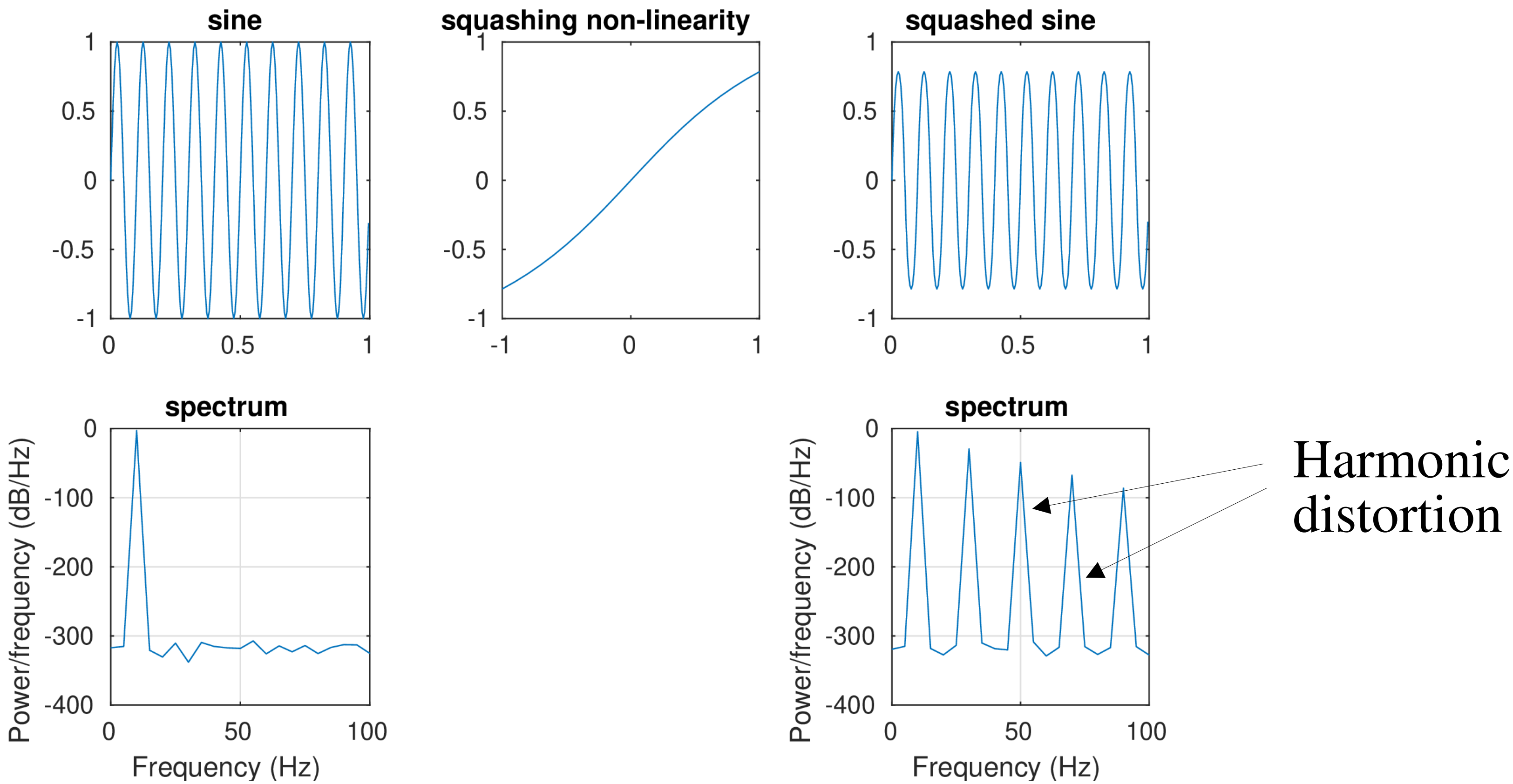

```
>> plot(-x(1,:),x(8,:))
```

```
>> plot(-x(1,:),x(2,:))
```

Common problem is limited dynamic range.

Linear, Stationary, Normal *- The stuff biology is not.*

Note that non-linearity can often be identified even in a 1D signal by it's **harmonic distortions**.




```
>> psd(sin(x))                >> psd(atan(sin(x))
Or with current matlab:
   periodogram(sin(x),[],fs/5,fs) % use fs=200
```

Linear, Stationary, Normal *- The stuff biology is not.*

Harmonic distortion explained ...

For example distortion of quadratic non-linearity leads to frequency doubling:

$$x(t) = \sin(\omega t)$$

$$y(t) = x^2(t) = \sin^2(\omega t) = \frac{1}{2} - \frac{1}{2}\cos(2\omega t)$$

Cubic leads triple frequencies:

$$y(t) = \sin^3(\omega t) = \frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)$$

General non-linearity contains all orders according to Taylor expansion:

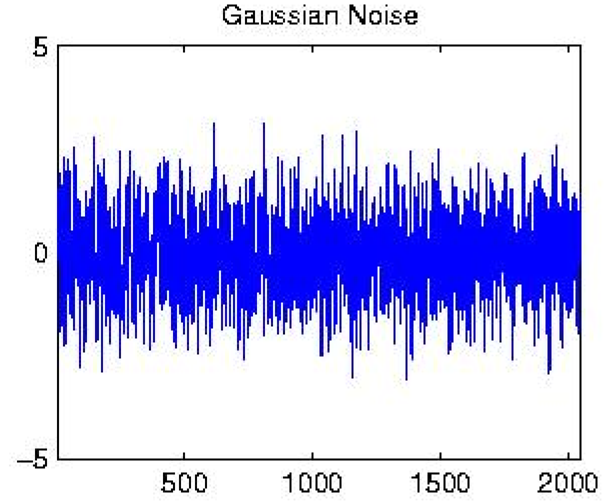
$$y = f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\frac{\partial^n f(x)}{\partial x^n} \right]_{x=0} x^n$$

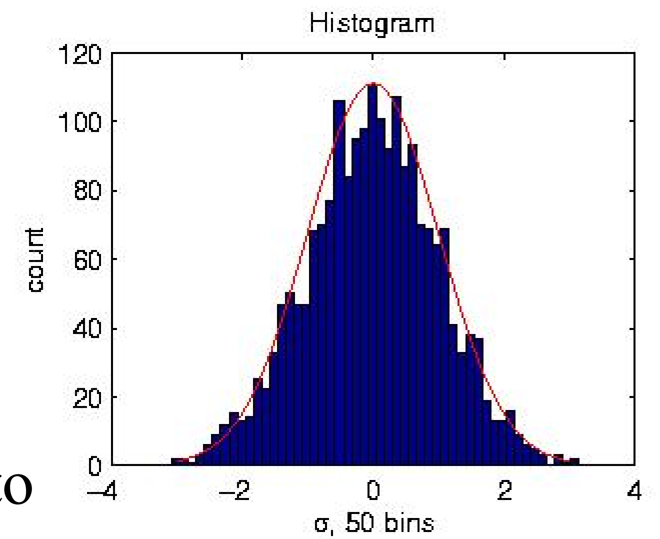
Linear, Stationary, **Normal** *- The stuff biology is not.*

Often '**normal**' distributions are assumed, i.e. Samples are **Gaussian distributed**.

Important because of many nice properties of the Gaussian probability density function (pdf):

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$





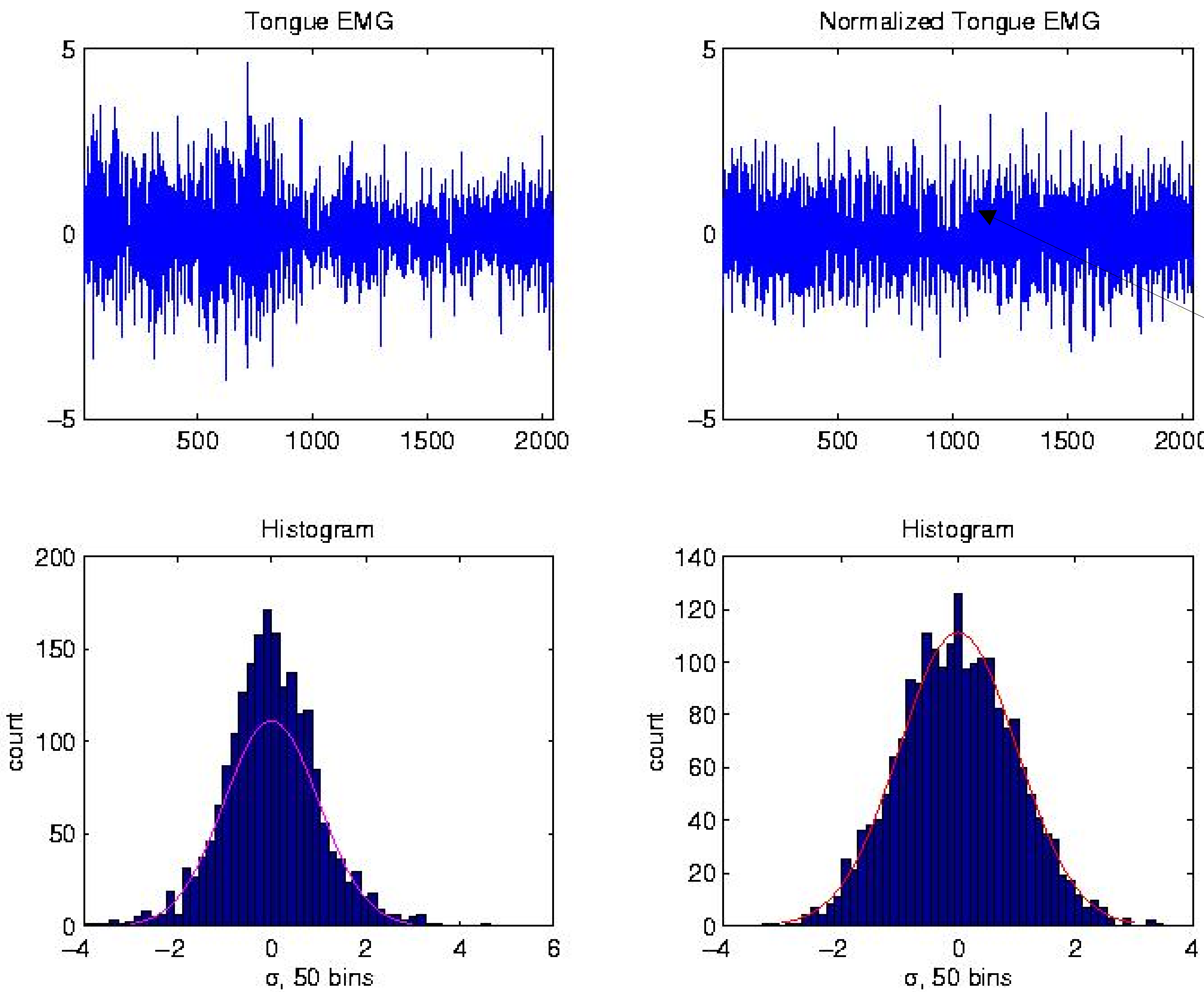

```
>> hist(randn(1000,1))
```

- Convolution of Gaussian remains Gaussian
- Product of Gaussian remains Gaussian
- Parameter σ easy to estimate.
- Leads to least squares optimization criteria
- Sums of many random variables converges to Gaussian, e.g. Brownian motion

Linear, Stationary, **Normal** *- The stuff biology is not.*

Unfortunately **many natural signals are NOT Gaussian.**

On the left is an example of Tongue electro-myogram (EMG):



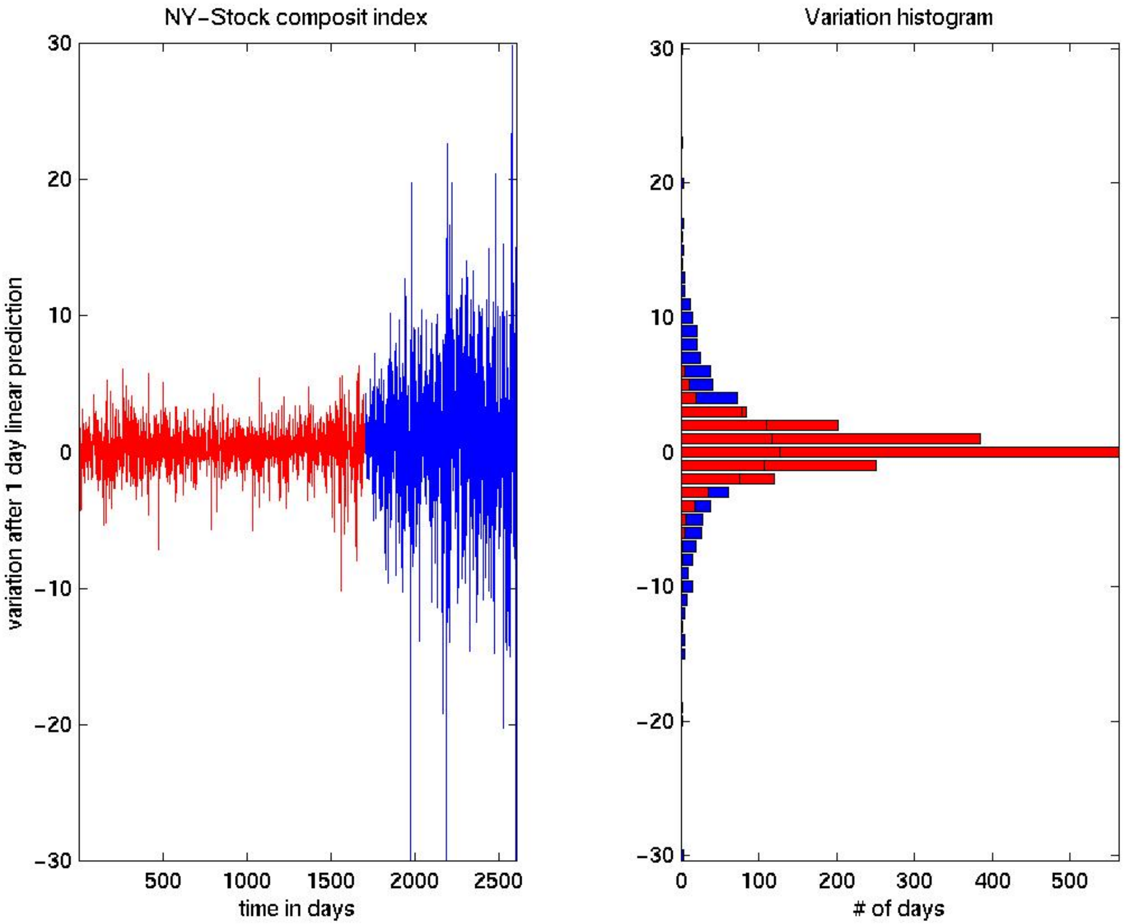
However, if we normalize by estimate of the local standard deviation:

```
s  =  std(x(i-10:i+10));
y(i)  = x(i)/s;
```

we obtain often something that is close to Gaussian.

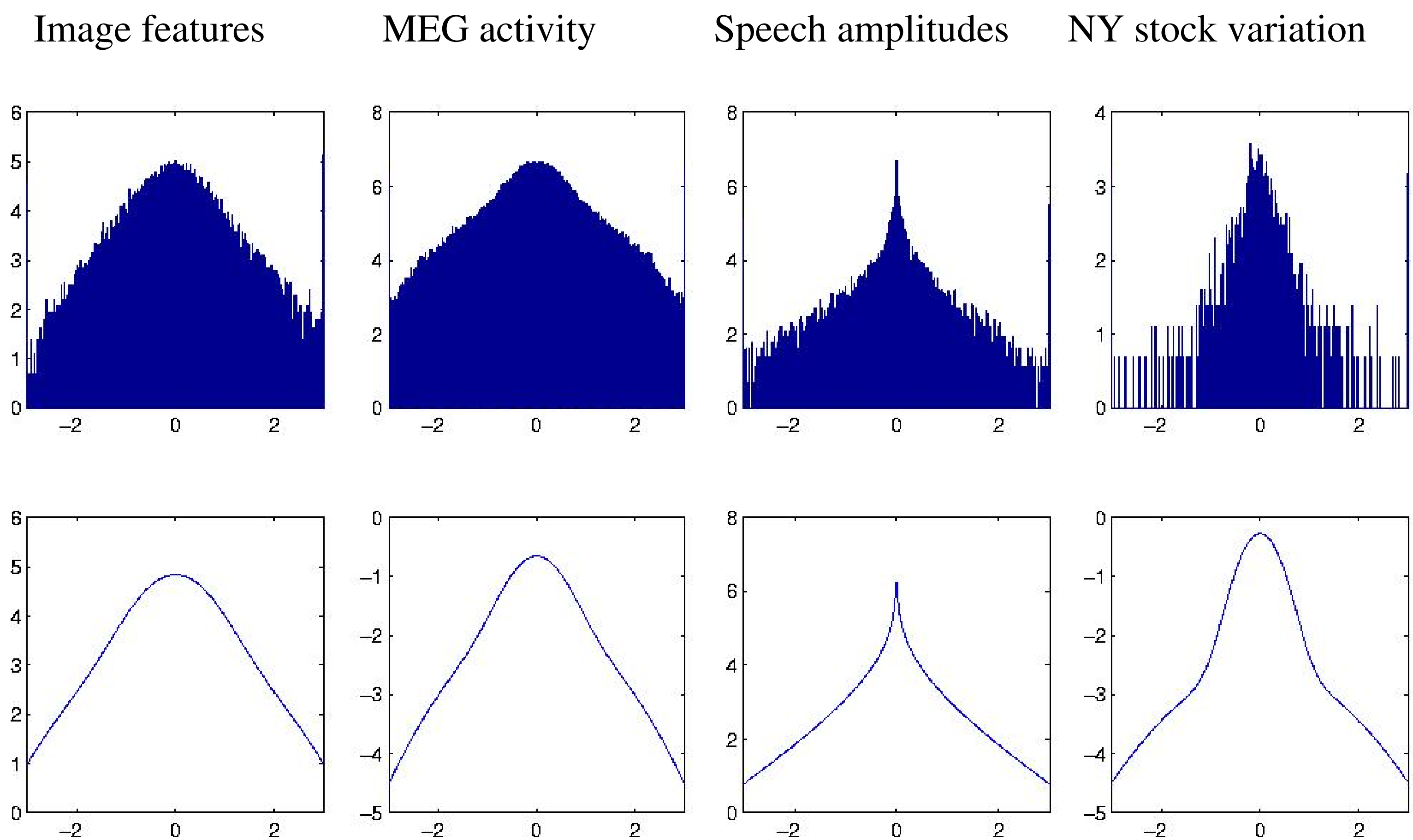
Linear, **Stationary, Normal** *- The stuff biology is not.*

The property of *heteroscedastisity* is often used in the context of financial time series. e.g. NY stock exchange index. It states that the signal is short term Gaussian with **time varying** standard deviation.



Linear, **Stationary, Normal** *- The stuff biology is not.*

There are **many non-stationary signals** that can be explained in first approximation as heteroscedastic. In multiple dimensions these signals are also known as *spherical invariant random processes*.



Linear, Stationary, Normal *- The stuff biology is not.*

- Many natural signals are **not stationary**, and **not normal**, and many systems are **not linear**.
- Analysis and signal processing is **OFTEN EASIER** if one can assume stationary, normal signals and linear systems.
- It is important to identify the nature of the signals and possibly apply preprocessing to make the assumptions simpler.
- Non-linearity may be identified simply by looking at scatter plots, or harmonic distortions if a strong oscillation is present (often 60Hz).
- Non-Gaussian properties can be identified by looking at histogram. We will use cummulants to asses 'normality' quantitatively.

All signal analysis starts by **LOOKING AT THE DATA**!

Grading

Assignment 1: Reproduce the four figures on slides 8, 9, 11, 12 from the raw data. Use the files eeg.mat and tongemg.mat.

For help on MATLAB run

```
>> demo
>> help
```

Useful functions

```
>> lookfor
>> whos
```

In particular, if you are new to matlab, please make substantial time available to run the demo programs which are a very good introduction to matlab: basic matrix operations, line plotting, matrix manipulations, 2-D plots, matlab language introduction, axis properties, graphs and matrices, and maybe some of the desktop environment demos as you see fit.

Grading

Good News - **No final nor midterm exam!**

Bad News - Assignments:

1. MATLAB programing
   - Turn in next week my email
   - Needs to run correctly 75% of the time for passing.
   - Needs to run perfectly 100% for the time for A+.
   - May have pop quizzes to test "undisclosed collaborations".
2. Proofs
   - Turn in next week
   - Easy, just to exercise the notation
3. Reading
   - Understand the subject and cover gaps
4. May have pop quizzes on reading and programming assignments.

Programming Assignments

- **If you copy code you will fail the course.**
- Assignments due in one weeks time. Submit per email **before class**.
- Submit **single executable file** called: first_last_number.m, all lower case e.g. john_smith_3.m for John's 3rd assignment. No figures, no text files, nothing except executable code.
- Your program must load all required data. Assume that data files are in current directory. All required data will be posted on the web.
- Include '`clear all, close all`' at the beginning of all programs.
- **Do not use upper case** letters for commands, e.g. Use `axis()` instead of '`AXIS()`'. They may work for you but they don't work for me!
- If you had help during your work, you **MUST** name your partner. **"Similar" submission are easy to spot.** *Undisclosed collaborations* receive 0 credit.
- The criteria for full credit should be clear. If not, please ask in class. Do not take chances by assuming that your work is "sort of correct".